Available online at www.sciencedirect.com
ScienceDirect
Cognition 107 (2008) 1–18
COGNITION
www.elsevier.com/locate/COGNIT

Parallel non-verbal enumeration is constrained by a set-based limit

Lisa Feigenson

Department of Psychological & Brain Sciences, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA

Received 11 July 2006; revised 5 July 2007; accepted 20 July 2007

Abstract

Adults can represent approximate numbers of items independently of language. This approximate number system can discriminate and compare entities as varied as dots, sounds, or actions. But can multiple different types of entities be enumerated in parallel and stored as independent numerosities? Subjects who were prevented from verbally counting watched an experimenter hide sequences of objects in two locations. The number of object types, which contrasted in category membership, color, shape, and texture, varied from 1 to 5, and object types were completely temporally intermixed. Subjects were then asked how many objects of each type were in each location. In three experiments, subjects successfully enumerated the objects of each type in each location when 1–3 types were presented, but failed with 4 or 5 types, regardless of the total number of objects seen. Thus, adults can perform simultaneous enumeration of multiple sets that unfold in temporally intermixed fashion, but are limited to 3 such sets at a time. Furthermore, they perform these parallel enumerations in the absence of training or instruction, and can do so for sets of objects that are hidden in distinct locations. The convergence of this 3-*set* capacity limit with the 3-*item* capacity limit widely observed in studies of working memory suggests that each enumeration requires a single slot in memory, and that storage in memory is required before enumeration can occur.

Keywords: Number; Enumeration; Sets; Working memory

E-mail address: feigenson@jhu.edu

0010-0277/$ - see front matter
doi:10.1016/j.cognition.2007.07.006

1. Introduction

Many species, including humans, monkeys, rats, and pigeons, represent numerical information without verbally counting. They can discriminate and compute over various visual and auditory entities (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004 for reviews) using representations that are abstract rather than modality-specific (Barth, Kanwisher, & Spelke, 2003; Barth, LaMont, Lipton, & Spelke, 2005; Feron, Gentaz, & Streri, 2006; Jordan & Brannon, 2006; Kobayashi, Hiraki, & Hasegawa, 2005). Furthermore, the neural underpinnings of this ability have begun to be identified. Human nonverbal enumeration is believed to rely on a dedicated region of the intraparietal sulcus (Ansari, Dhital, & Soon, 2006; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Pinel, Piazza, LeBihan, & Dehaene, 2004). When this area is engaged, performance shows a characteristic pattern of error that increases with the size of the target array (Barth et al., 2003; Cordes, Gelman, Gallistel, & Whalen, 2001; Lipton & Spelke, 2003; Moyer & Landauer, 1967; Whalen, Gallistel, & Gelman, 1999). This pattern has led to the predominant view that number is represented nonverbally as a series of mental magnitudes whose degree of precision is inversely related to the size of the to-be-represented quantity.

Activation of this approximate number system has been demonstrated for entities ranging from 2-dimensional dots to auditory tones to lever presses. However, in all of these cases human and animal subjects have been required to enumerate just a single set of entities at a time. Even when comparing two arrays presented in different modalities, subjects first enumerated one array and then enumerated the next. But daily life presents situations that merit multiple enumerations at the very same time. For example, a group of cooperative animals on a hunt might determine the ratio of their group members to that of their prey in order to decide whether to attack or move on. In such a situation, one could enumerate first the predators, store the result in memory, and then enumerate the prey. But could one simultaneously enumerate both of these as independent sets?

One recent demonstration has shown that human adults indeed can represent the numerosities of several sets at once (Halberda, Sires, & Feigenson, 2006). Observers saw dot displays comprised of 1–6 colors, with varying numbers of dots in each color subset and subsets completely spatially intermixed. The displays appeared for 500 ms and were forward- and backward-masked. On some of the trials, observers were told before the onset of the display which color they were to enumerate (Probe Before trials). On other trials they were not told until after the display had disappeared (Probe After trials). Comparing the accuracy of enumeration on these two types of trials revealed the number of colored subsets observers could simultaneously enumerate in a single flash. When enumeration on Probe After trials was just as accurate as on Probe Before trials, observers had to have stored the approximate numerosities of all of the color subsets in the array. When performance was significantly worse on Probe After than Probe Before trials, this indicated that the number of color subsets had exceeded the number of enumerations that could be simultaneously performed. This calculation revealed each observer's enumeration capacity. On average, observers could enumerate 3 spatially-intermixed sets. However, even despite the brief, 500-ms display time, it was not possible to definitively conclude that adults were actually performing multiple enumerations in parallel. Another possibility is that they first enumerated one set, very rapidly stored the result in memory, and then enumerated the other set(s). Evidence that numerosity-tuned neurons in monkey cortex begin firing an average of only 120 ms after the onset of a stimulus containing their preferred numerosity (Nieder, Freedman, & Miller, 2002) lends some indirect support to this possibility.

Still, the claim that adults can likely enumerate multiple sets at once raises several interesting questions. First, the colored subsets in the displays used by Halberda et al. were simultaneously visible – all of the subsets appeared on screen at the same time. The challenge of multiple enumeration is highlighted when stimuli are not simultaneously visible in this way but rather appear sequentially – for instance, when many red marbles and many blue marbles are dropped into a jar in intermixed order. In this case neither set of marbles is simultaneously visible in its entirety, rendering useless the strategy of first enumerating one set and then the other. Indeed, it seems that without keeping track of both sets in parallel as they accumulate, enumeration would be impossible. Recent work suggests that the enumeration of simultaneously-presented and sequentially-presented stimuli are in fact subserved by different populations of neurons, at least in primates (Nieder, Diester, & Tudusciuc, 2006). It is therefore unclear to what extent simultaneous- and sequential-enumeration share a common set of properties, including the possibility of enumerating multiple sets. Can adults perform multiple enumerations on sets that accumulate over time and thus overlap temporally, rather than spatially? If so, this would be stronger evidence for a genuine ability to perform multiple enumerations in parallel.

Furthermore, positive evidence for the enumeration of multiple sequentially-presented sets would help to pinpoint the locus of the 3-set limit observed by Halberda et al. (2006). One possibility is that enumeration capacity is constrained by attention: Only up to 3 sets can be enumerated because only up to 3 sets can be visually attended in parallel (see Pylyshyn & Storm, 1988; for evidence that no more than 3–4 *individual objects* can be attended in parallel). Another possibility is that enumeration capacity is constrained by memory: Only up to 3 sets can be enumerated because only up to 3 sets can be stored in memory in parallel (see Alvarez & Cavanagh, 2004; Luck & Vogel, 1997; Xu & Chun, 2006 for evidence that no more than 3 *individual objects* can be stored in parallel). Both of these options are consistent with the results of Halberda et al., because all of Halberda's sets were simultaneously visible in the form of spatially-intermixed dot arrays. If we also observe the 3-set enumeration limit with sequentially-presented, temporally-intermixed dot displays, it would favor the explanation that capacity is limited by memory rather than by attention, since no more than 1 object need be attended at a time.

Earlier work has asked whether adults can represent sequentially-presented sets (Erlick, 1964; Karsh, 1969; Yntema & Mueser, 1960; Yntema & Schulman, 1967). For example, Monty, Taub, and Laughery (1965) flashed sequences containing varying numbers of tokens of 2–4 letter types, and asked subjects to write down how many tokens of each letter comprised each sequence. Monty et al. found that subjects produced fairly accurate responses, and that performance decreased as the number of letter types, total number of tokens, and speed of presentation increased. However, these findings do not reveal whether the approximate number system can enumerate multiple sets in parallel because subjects were allowed to verbally count the letters (and almost all subjects reported doing so). This leaves open the possibility that language is required to maintain multiple tallies. Furthermore, since Monty et al. report the total number of counting errors collapsed across all letter types, rather than separately reporting the number of errors made within each letter type, it is hard to know how many distinct letter types subjects were able to accurately represent.

A second open question is whether the ability to perform multiple enumerations is the result of extensive training. The observers in the studies by Halberda et al. (2006) and Monty et al. (1965) participated in several hundred trials each, and were explicitly told to enumerate. Therefore, it remains unclear to what extent multiple-set enumeration is spontaneously deployed, or rather develops in response to explicit instruction and/or practice.

Finally, the discovery of a 3-*set* limit on multiple enumeration is intriguing in that it converges with the 3-*object* limit observed in studies of visual attention (Pylyshyn & Storm, 1988) and working memory in adults (Alvarez & Cavanagh, 2004; Cowan, 2001; Luck & Vogel, 1997; Xu & Chun, 2006) and infants (Feigenson, Carey, & Hauser, 2002; Feigenson & Carey, 2003, 2005). In these studies, both adults and infants were limited to tracking approximately 1, 2, or 3 objects at a time; subjects failed with larger numbers. But the evidence from infants suggests that, at least in some situations, the 3-item limit applies locally rather than globally. For instance, infants who had to remember the number of crackers hidden at each of two locations successfully chose the greater quantity when presented with 1 cracker in the left location and 2 crackers in the right location (a choice between 1 vs. 2 crackers), and also succeeded with 2 vs. 3 crackers, but failed with 3 vs. 6, 3 vs. 4, 2 vs. 4, and even 1 vs. 4 crackers (Feigenson et al., 2002; Feigenson & Carey, 2005). Note that the total number of crackers involved in a 2 vs. 3 comparison is the same as that involved in a 1 vs. 4 comparison, but the 2 vs. 3 comparison contains 3 or fewer objects at each location while the 1 vs. 4 comparison exceeds infants' 3-item limit at one of the locations. Infants' success at 2 vs. 3 and failure at 1 vs. 4 shows that infants were limited to tracking up to 3 hidden objects per location, rather than 3 hidden objects in total. Might the limit on adults' ability to perform multiple enumerations be similarly constrained to allow up to 3 enumerations at a time *at any given location*?

The present studies sought to confirm that adults can represent the numerosity of several sets at once and asked: (1) Whether subjects can perform multiple enumerations of sequentially-presented, temporally intermixed items, as they can over simultaneously-presented items; (2) whether multiple enumeration is spontaneously performed without training or explicit instruction; and (3) whether subjects can represent the numerosity of multiple sets at two distinct locations.

These questions were addressed using a modification of the procedure that has been used with infants. Adult subjects watched an experimenter sequentially place a series of objects into one bucket, and then another series of objects into another bucket (see Feigenson & Carey, 2005; Feigenson et al., 2002). Verbal shadowing during the presentation prevented subjects from counting the items. After the presentation, subjects were asked which bucket contained more objects of a given type and approximately how many objects of that type were in each bucket. Critically, the number of object types varied across subjects. In the 1-Type Condition, subjects saw two different quantities of just a single object type (e.g., 5 candies hidden in one bucket and 10 candies in the other). In the 2-Type Condition, subjects saw two different types of objects hidden in each bucket (e.g., 5 candies temporally intermixed with 10 toy pigs in one bucket and 10 candies intermixed with 5 pigs in the other). Subjects saw 1, 2, 3, 4, or 5 object types, and objects were always presented in a pseudo-random order that intermixed the different object types within each bucket. The ratios between the two hidden quantities (1:2 in Experiment 1; 2:3 in Experiment 2) were chosen because they are known to be discriminable by adults performing nonverbal enumeration (Barth et al., 2003; Whalen et al., 1999). It was expected that adults would successfully enumerate in the 1-Type Condition, consistent with previous results, but unclear whether they would succeed in the 2-, 3-, 4-, and 5-Type Conditions.

2. Experiment 1

2.1. Method

Eighty adults (51 female; mean age 19.7 years) who were native speakers of English received course credit, candy, or payment in exchange for participation. Subjects sat at a table across from the experimenter. First they were instructed in verbal shadowing. They engaged in approximately 1 min of practice repeating a random letter sequence that had been recorded by a female speaker (0.77 letters/s). Subjects were told to repeat each letter immediately upon hearing it, and to do so without stopping to correct any errors. All subjects were judged to have adequately engaged in this shadowing task, and so continued on to the next phase of the experiment.

Next, the experimenter told subjects that they would see a short sequence of events while verbally shadowing, after which they would be asked simple questions about the events. Subjects resumed shadowing and saw two short "practice events" in which the experimenter placed 3 toys under 3 cups, moved the cups around the table top in front of the subject and then revealed them, then hid 3 balls behind 2 small screens and revealed them. These practice events lasted approximately 1 min in total. Subjects watched these events while continuously shadowing, and were not told that these were practice events.

For the final critical event, the experimenter placed 2 opaque buckets 80 cm apart, then sequentially hid different quantities of objects in each. For example, subjects in the 1-Type Condition saw 5 toy pigs placed one at a time into one bucket and 10 in the other. The experimenter took the objects from a container hidden under the table and waved each object above the bucket before placing it inside, making sure subjects were attending. One object was placed approximately every 2 s. The experimenter finished placing all of the objects in one bucket before moving on to the second bucket.

For each object type, there were always 5 in one bucket and 10 in the other (Table 1). The number of object types presented varied by condition, with 16 subjects (approximately equally balanced by sex) in each condition. The objects were approximately 1–2 in. long and differed in category membership, color, shape, and texture. They consisted of pink rubber pigs, green Starburst candies, black AA batteries, blue plastic poker chips, and white cottonballs. In the conditions in which multiple object types were presented, objects were presented in a fixed pseudo-random order such that the different object types were completely temporally intermixed. Appendix A lists the orders that were used.

After placing all of the objects, the experimenter turned off the shadowing recording and asked subjects a series of questions. For each object type presented, the experimenter asked: (1) Please point to the bucket that has more (X). (2) How many (X) do you think are in that bucket? (3) How many are in the other bucket? Subjects were asked these 3 questions about the first object type, then about the second object type, etc. If subjects guessed that the buckets contained equal numerosities of a given type (e.g., "There are 5 pigs in the left bucket and 5 in the right"), they were told that the buckets contained unequal numbers and were asked to adjust their answers accordingly. If subjects expressed uncertainty, they were told to "Give your best guess." After subjects had answered these questions for all object types presented, the experimenter finished by asking: How did you know how many objects were in each bucket, and did you explicitly count?

Which bucket contained more of each object type, which bucket the presentation began in, and in which order the object types were queried by the experimenter were all counterbalanced across subjects. Experiment sessions were videotaped to ensure that subjects had adequately verbally shadowed throughout the duration. All subjects were judged to have done so.

2.2. Results

The first question was whether subjects in the different conditions correctly chose which bucket contained more objects. Although it is possible to analyze subjects' total correct answers collapsed across all of the object types queried (ranging from 1 to 5 types, depending on condition), asking the question in this way introduces several unwanted sources of variance. First, if subjects' memory of the object sequences decayed at all over time, this would be reflected in higher accuracy for the first object type queried and lower accuracy for the last object types. Second, subjects might have adjusted their guesses for any given object type based on the answers given for the object types previously queried[1]. Therefore, analyzing responses for the first object type queried provides a more accurate reflection of subjects' abilities (recall that which

[1] Subjects' responses suggested that both of these may have affected their numerical judgments. Accuracy was slightly higher on the first object type queried than on the last type queried in 4 out of the 4 conditions involving more than one type. Additionally, some subjects attempted to revise their numerical estimations when queried about multiple object types (for example, one subject initially answered that there were 6 candies in the left-hand bucket but then corrected his estimate, saying "Six Starbursts...no, I guess 12. There were more Starbursts than batteries, so 12.").

Table 1
Distribution of object types in each condition of Experiments 1–3

Experiment	Condition	Bucket A	Bucket B
1	1-Type	5 toy pigs	10 toy pigs
	2-Type	5 toy pigs, 10 candies	10 toy pigs, 5 candies
	3-Type	5 toy pigs, 10 candies, 10 batteries	10 toy pigs, 5 candies, 5 batteries
	4-Type	5 toy pigs, 10 candies, 10 batteries, 5 poker chips	10 toy pigs, 5 candies, 5 batteries, 10 poker chips
	5-Type	5 toy pigs, 10 candies, 5 batteries, 10 poker chips, 10 cottonballs	10 toy pigs, 5 candies, 10 batteries, 5 poker chips, 5 cottonballs
2	1-Type	6 toy pigs	9 toy pigs
	2-Type	6 toy pigs, 9 candies	9 toy pigs, 6 candies
	3-Type	6 toy pigs, 9 candies 9 batteries	9 toy pigs, 6 candies, 6 batteries
	4-Type	6 toy pigs, 9 candies 9 batteries, 6 poker chips	9 toy pigs, 6 candies, 6 batteries, 9 poker chips
	5-Type	6 toy pigs, 9 candies 9 batteries, 6 poker chips, 9 cottonballs	9 toy pigs, 6 candies, 6 batteries, 9 poker chips, 6 cottonballs
3	3-Type	7 toy pigs, 14 candies, 7 batteries	14 toy pigs, 7 candies, 14 batteries
	4-Type	4 toy pigs, 8 candies, 4 batteries, 8 poker chips	8 toy pigs, 4 candies, 8 batteries, 4 poker chips

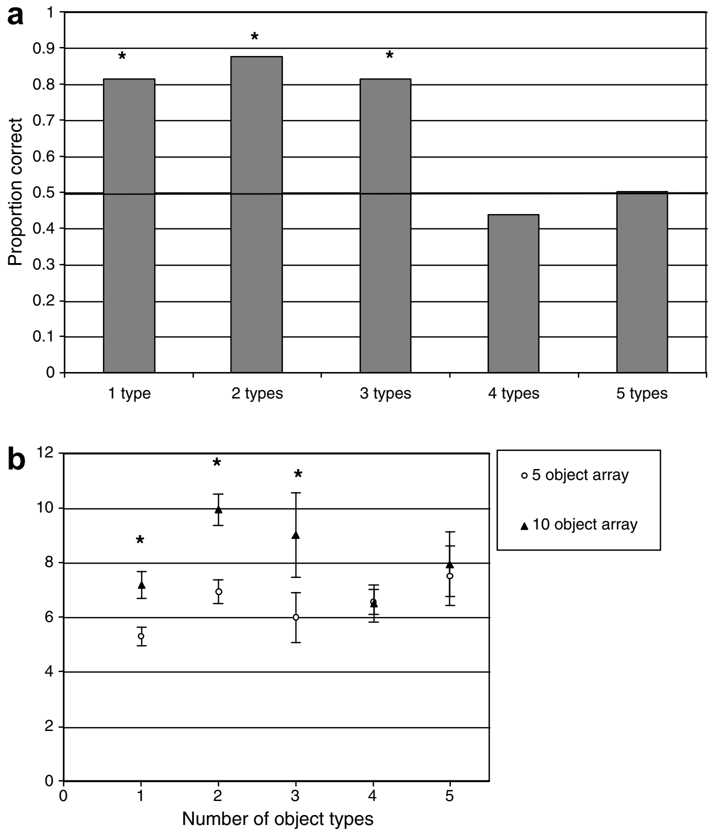

Fig. 1. (a) Proportion of correct responses indicating which bucket contained more objects for the first object type queried in Experiment 1. The solid line indicates chance performance. (b) Subjects' numerical estimates for the 5- and 10-object arrays in Experiment 1, shown by condition. Bars indicate standard error.

object type was queried first was counterbalanced across subjects and that subjects did not know which type(s) they would be asked about).

Analysis of responses to the first object type queried revealed that subjects in the 1-, 2-, and 3-Type Conditions were all above chance in choosing which bucket held more objects: 1-Type: 13/16 correct, $p < .05$; 2-Type: 14/16 correct, $p < .01$, and 3-Type: 13/16 correct, $p < .05$, all 2-tailed sign tests (Fig. 1a). In contrast, subjects in the 4- and 5-Type Conditions were at chance: 4-Type: 7/16 correct, $p = .804$; 5-Type: 8/16 correct, $p = .99$. Since subjects did not know which object type they would be queried about first, this performance pattern suggests that subjects were able to enumerate up to 3 object types at a time[2]. When 4 or 5 types were presented, performance completely fell

[2] Analyzing responses across all trials, rather than just responses on the first object type queried, yielded the same general pattern. Collapsed across all types queried, the proportion of correct responses was: .81 (1-Type Condition), .91 (2-Type Condition), .69 (3-Type Condition), .55 (4-Type Condition), and .59 (5-Type Condition).

apart and subjects were unable to enumerate *any* of the types. This catastrophic failure parallels that observed when infants in a similar paradigm see 4 or 5 individual objects hidden – for example, after seeing 1 object hidden in one bucket and 4 in the other, infants chose randomly between the two locations (Feigenson et al., 2002; Feigenson & Carey, 2005). This convergence suggests that whether tracking individual *objects* or individual object *types*, both adults and infants are limited to encoding information from no more than 3 items at once. Furthermore, at least in the tasks that have been used with adults (in the present experiments) and with infants (in Feigenson et al., 2002 and Feigenson & Carey, 2003), when more than 3 items are presented subjects lose track of *all* items rather than just the items presented after memory capacity has been reached.

Because subjects also reported how many objects of each type were in each bucket, it is possible to ask whether numerical estimates correctly approximated the target number of objects hidden, and to determine whether their estimates of the two numerical quantities differed significantly from one another. Producing reliably higher estimates for the 10-object sequence than for the 5-object sequence would be further indication that subjects successfully enumerated the quantities. Furthermore, if subjects were relying on their approximate number system rather than verbally counting, their answers should not fall exactly on the target number. Consistent with these predictions, subjects were not only successful at recognizing the ordinal relationship between the presented quantities, but also were fairly accurate in their numerical approximations, although their answers were not exactly correct (Fig. 1b). Estimates of the two numerosities differed significantly when 1, 2 or 3 types were presented, but not when 4 or 5 types were presented (Fig. 1b). Subjects in the 1-, 2-, and 3-Type Conditions successfully produced higher estimates for 10-object sequences than for 5-object sequences, as revealed by within-subject *t*-tests for each condition: 1-Type: $t(15) = 3.429$, $p < .005$; 2-Type: $t(15) = 6.445$, $p < .001$; 3-Type: $t(15) = 2.748$, $p < .02$. In contrast, subjects in the 4- and 5-Type Conditions did not produce reliably different numerical estimates for the two sequences: 4-Type: $t(15) = -.112$, $p = .912$; 5-Type: $t(15) = .492$, $p = .630$. This further confirms that subjects were unable to track numerosity when 4 or 5 object types were seen. Additionally, the finding that subjects produced approximately correct numerical responses argues against the possibility that their judgments were based on timing rather than number. Timing was confounded with object number, in that for any given object type it always took longer to present the sequence containing the greater number of object tokens. It is therefore conceivable that subjects in these conditions tracked up to 3 temporally intermixed presentation durations (see Meck & Church, 1983), in which case the present results would have implications for limits on the number of durations that can be represented, rather than for the number of enumerations that can be represented. However, the fact that subjects were able to report approximately how many discrete objects were in each location suggests that they had access to numerical information about the events, making the duration account less plausible.

None of the subjects reported explicitly counting to obtain their numerical estimations, although a few said they had tried but lost track after the first few objects.

When asked how they arrived at their estimates, typical responses were that they "weren't sure," "just had a feeling," or "just saw more objects on that side than the other."

Thus, Experiment 1 demonstrates that adults can non-verbally enumerate multiple temporally-intermixed sets in parallel. Moreover, they are limited to enumerating no more than 3 sets at any one time. This ability appears to be spontaneously employed, as subjects had no idea that they were in a number-task, much less that they would be asked to track the numerosities of multiple sets at once. And finally, subjects successfully represented the approximate numerosity of up to 3 *sets of objects* in each of two hiding locations, paralleling infants' performance at tracking *individual objects* in two hiding locations. Given the surprising nature of these results, a second experiment was conducted to confirm this effect. Experiment 2 aimed to replicate Experiment 1 using the more difficult ratio of 2:3 instead of 1:2 (i.e., 6 vs. 9 objects rather than 5 vs. 10). If adults can indeed enumerate up to 3 sets of items at a time, then they should be able to do so with a range of numerical values so long as those values are discriminable by the approximate number system.

3. Experiment 2

3.1. Method

One hundred adults (63 female; mean age 21.2 years) who were native speakers of English received course credit, candy, or payment in exchange for participation. The stimuli and procedure were identical to those of Experiment 1, except that the presentation involved intermixed sets of 6 and 9 objects rather than 5 and 10 objects (Table 1). Different object presentation orders were used (see Appendix A) to ensure that the results did not depend on seeing any particular sequence.

3.2. Results

Experiment 2 replicated the findings of Experiment 1. As in Experiment 1, subjects in the 1-, 2-, and 3-Type Conditions were above chance in choosing which bucket held more objects for the first object type they were queried about: 1-Type: 15/20 correct, $p < .05$; 2-Type: 16/20 correct, $p < .02$, and 3-Type: 18/20 correct, $p < .001$, all 2-tailed sign tests (Fig. 2a). In contrast, subjects in the 4- and 5-Type Conditions were at chance: 4-Type: 10/20 correct, $p > .99$; 5-Type: 10/20 correct, $p > .99$.[3]

Analysis of subjects' estimates of the number of objects in each bucket again shows that they successfully formed numerical approximations but were unable to keep exact counts, and also that they produced reliably higher estimates for the

[3] Analyzing subjects' total correct responses yielded the same general pattern. Here, the proportion of correct responses was: .75 (1-Type Condition), .83 (2-Type Condition), .75 (3-Type Condition), .55 (4-Type Condition), and .46 (5-Type Condition).

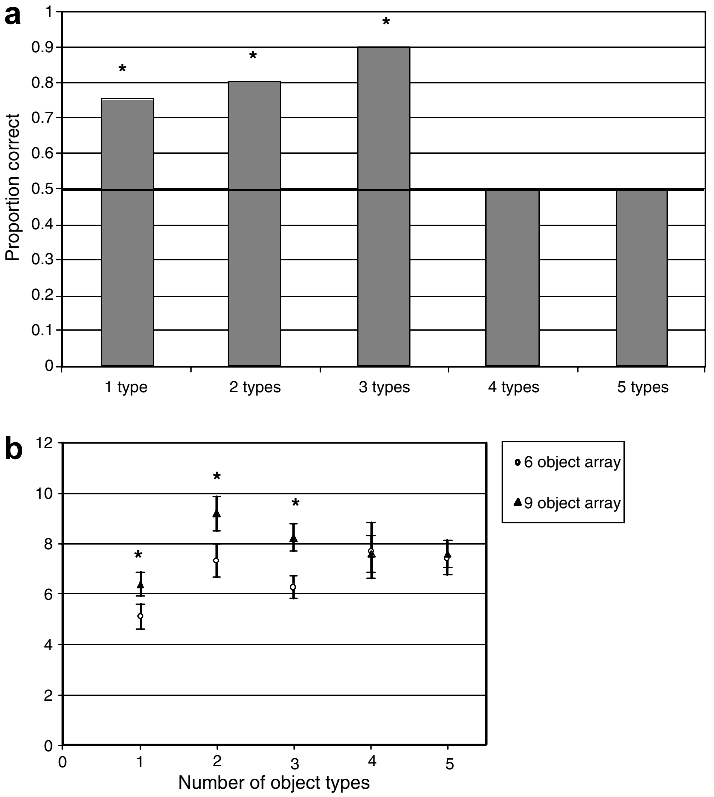

Fig. 2. (a) Proportion of correct responses indicating which bucket contained more objects for the first object type queried in Experiment 2. The solid line indicates chance performance. (b) Subjects' numerical estimates for the 6- and 9-object arrays in Experiment 2, shown by condition. Bars indicate standard error.

9-object sequence than for the 6-object sequence when presented with 1, 2, and 3 object types, but not with 4 or 5 types. This is revealed by within-subject *t*-tests on subjects' numerical estimates of the 6- versus 9-object sequences for each condition (Fig. 2b): 1-Type: $t(19) = 2.16$, $p < .05$; 2-Type: $t(19) = 2.83$, $p < .01$; 3-Type: $t(19) = 4.79$, $p < .01$. 4-Type: $t(19) = .16$, $p = .87$; 5-Type: $t(19) = .21$, $p = .83$. This further confirms that subjects were unable to track the number of objects involved when 4 or 5 object types were seen.

4. Experiment 3

One concern with subjects' failure with 4 and 5 object types in Experiments 1 and 2 is that these conditions involved more *total* objects than the 1-, 2-, and 3-Type conditions. The greater presentation length and complexity in the 4- and 5-Type

Conditions could conceivably have caused subjects to lose motivation or become overwhelmed over the course of the presentation. Might this, rather than any limit on the number of object types that can be enumerated in parallel, have led to their failure? The finding that performance plummeted abruptly when more than 3 object types were presented mitigates against such an explanation, since the gradual increase in presentation length and complexity as the number of object types increased would more plausibly lead to an accompanying gradual decrement in performance. However, Experiment 3 sought to replicate the performance break between 3 and 4 object types while explicitly ruling out total presentation duration and complexity as causes.

4.1. Method

Thirty-two adults (22 female; mean age 19.5 years) who were native speakers of English received course credit, candy, or payment in exchange for participation. The stimuli and procedure were identical to those of Experiments 1 and 2. The 16 subjects assigned to the 3-Type Condition saw toy pigs, candies, and batteries. The ratio between the presented quantities was the same as in Experiment 1 (1:2), but the number of objects was increased to 7 and 14 objects of each type (Table 1). This allowed the total number of objects in the 3-Type Condition to increase from 45 in Experiment 1 to 63 in the present experiment. The 16 subjects assigned to the 4-Type Condition saw toy pigs, candies, batteries, and poker chips. Again, the ratio between the presented quantities was 1:2, but the number of objects was reduced to 4 and 8 objects of each type. This allowed the total number of objects in the 4-Type Condition to decrease from 60 in Experiment 1 to 48 in the present experiment.

Critically, in Experiment 1 the 4-Type Condition contained more total objects (60) than the 3-Type Condition (45), but in the present experiment the 3-Type Condition contained more total objects (63) than the 4-Type Condition (48). If performance is determined by presentation length or complexity, subjects should now do worse in the 3-Type Condition. If instead performance is determined by the number of object types presented, subjects should again do worse in the 4-Type Condition.

4.2. Results

As predicted, subjects in the 3-Type Condition were above chance in choosing which bucket held more objects for the first object type queried, 13/16, $p < .05$, two-tailed sign test. In contrast, subjects in the 4-Type Condition were at chance, 9/16, $p = .804$, two-tailed sign test (Fig. 3a). In addition, subjects' numerical estimates were reliably higher for the greater quantity presented than for the smaller quantity in the 3-Type Condition, but not in the 4-Type Condition (Fig. 3b). This replicates the results of Experiments 1 and 2, despite the fact that here the 3-Type Condition involved more total objects, a longer total presentation duration, and greater presentation complexity. This is confirming evidence that success in this task depends on the number of object *types*, not on the total number of objects presented.

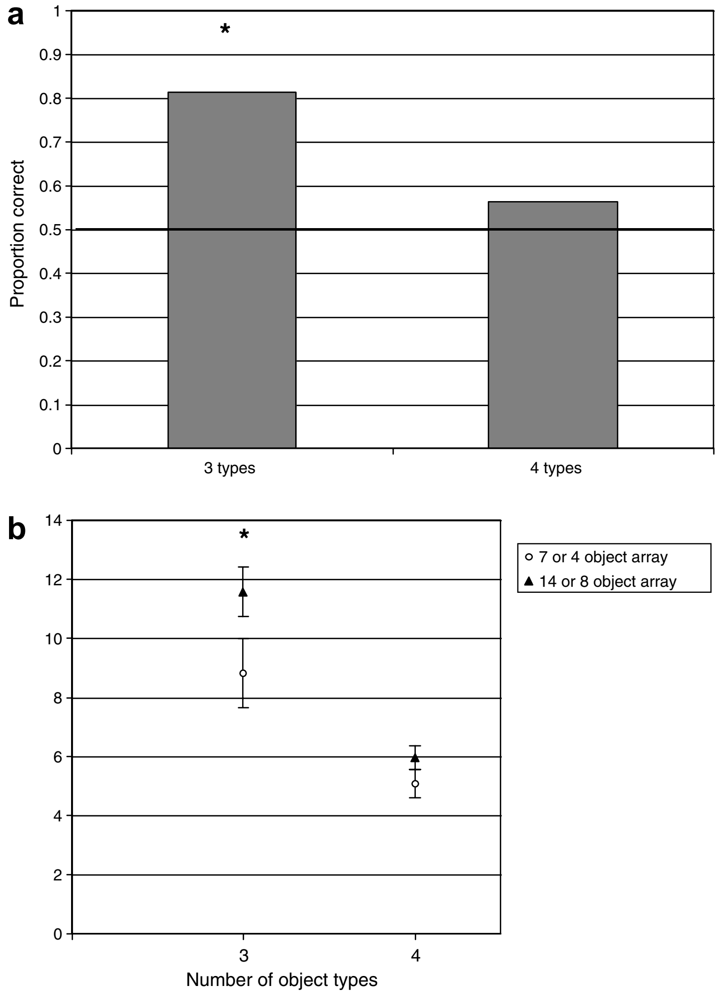

Fig. 3. (a) Proportion of correct responses indicating which bucket contained more objects for the first object type queried in Experiment 3. The solid line indicates chance performance. (b) Subjects' numerical estimates for the arrays in Experiment 3, shown by condition. Bars indicate standard error. Subjects in the 3-Type condition saw 7 vs. 14 objects and subjects in the 4-Type condition saw 4 vs. 8 objects.

5. General discussion

The present research confirms that adults can use their non-verbal approximation system to represent the numerosities of several sets at a time. Furthermore, they can do so for up to 3 sets, but no more. This sequence of studies thus replicates the limit on multiple enumeration observed by Halberda et al. (2006), and also addresses several important questions regarding the nature of this ability.

First, subjects were able to enumerate multiple sets of items even when those sets appeared sequentially rather than simultaneously. More impressively, the individual objects comprising the sets were seen in temporally-intermixed order such that subjects never knew when the final object for any given set had been presented. This means that all of the sets had to be actively maintained and incremented throughout the entirety of the presentation. This new finding that multiple enumerations can be performed over sequences that unfold over time suggests that animals can use their system of numerical approximation to track a wide variety of entities, including those that are temporally interspersed. These results can also help to identify the locus of the 3-item limit on multiple enumeration observed in the present experiments and in those of Halberda et al. (2006). In the current task only one item was ever visible at a time, such that attention was not deployed to multiple sets at once. That subjects still showed a 3-set capacity limit, despite the fact that attention was not taxed, suggests that the number of enumerations that could be performed was constrained by limits on working memory, rather than by limits on attention.

Second, this series of experiments shows that the ability to perform multiple enumerations does not rely on training or practice. Adults spontaneously enumerated up to 3 interspersed sequences in a single-trial task that they never heard described as numerical in nature. Indeed, many subjects expressed surprise and even dismay upon being asked how many objects had been presented, saying that they had no idea that they would be queried about the number of items they had seen. Despite this low degree of confidence, subjects were in fact quite accurate at judging the numerosity of up to 3 sets at a time even though they had not explicitly attempted to do so.

Finally, the present experiments show that adults can represent numerosities across multiple locations. Adults were able to compare the numerosity of up to 3 sets of objects hidden in each of two buckets, just as infants were able to track up to 3 individual crackers hidden in two buckets (Feigenson & Carey, 2003; Feigenson et al., 2002). For both adults and infants, the 3-item limit appears to constrain the number of sets of objects *or* the number of individual objects that can be represented at each hiding location rather than in the scene as a whole. This convergence strengthens the parallels between tasks tapping the abilities of infants and those of adults, as well as the parallels between capacity for remembering individual objects and capacity for remembering sets of objects. The commonalities between representing individual objects and representing sets demonstrate the remarkable flexibility of working memory, which appears to be equally adept at storing either level of representation. Both individual objects and sets of objects can be stored and compared in memory, both are subject to a 3-item capacity limit, and both are locally limited (by the number of items per location, rather than by the number of items in the entire scene). For working memory, a set functions as an individual just as an object does.

This last result has important implications for the two mental systems posited to underlie non-verbal number representation. Early debate centered on whether the approximate number system or a separate, item-based system for representing distinct individuals in working memory (often called the object-file or object tracking system, Scholl, 2001; Simon, 1997; Uller, Huntley-Fenner, Carey, & Klatt, 1999) was more likely responsible for adults' and infants' performance. More recent views offered that both systems are in place, and may operate under different conditions (Feigenson et al., 2004; Xu, 2003). The present work adds to this dialogue by suggesting an *integration* of the two systems. In order for the approximate number system to output a numerical representation, the referent set must first be attended and held in working memory before numerical information can be extracted. Limits on the number of items that can be stored in memory constrain the number of sets that can be stored, and hence the number of approximate enumerations that can be performed. Thus, it appears that both a system for representing distinct individuals in memory (an object-tracking system, or a slot-limited working memory) and a system for approximating numerosity are needed in order to recognize the number of items comprising a set. Up to about 3 sets may be held in memory, therefore up to 3 sets can be enumerated at once.

This integrated approach offers a new view on quantity representation. Rather than asking whether the object-file/working memory system *or* the approximate number system is activated in a given task, it appears that the approximate number system depends on memory for its input. Thus, the present experiments demonstrate how studying both the capacities and the limitations of numerical performance can lead to a better understanding of the ways in which several representational systems integrate to produce a unified representation of number.

Acknowledgements

I thank Zebib Abraham, Caroline Dixon, Catherine Juravich, Cecily Naron, and Julia Tracey for help in data collection; Susan Carey for support and discussion; and Justin Halberda for critical discussion of the experimental design and for comments on an earlier version of this manuscript. I also thank Brian Scholl and two anonymous reviewers for their input.

Appendix A

Order of object types in each experimental condition of Experiments 1–3

Experiment	Condition	Bucket A	Bucket B
1	1-Type	p,p,p,p,p	p,p,p,p,p,p,p,p,p,p
	2-Type	c,p,c,p,p,c,c,p,c,c,c,p,c,c,c	p,c,p,p,c,p,p,c,p,c,p,c,p,p,p
	3-Type	c,b,p,b,c,c,p,b,c,b,c,p,c,b,p,b,c,c,b,b,c,p,b,c,b	c,p,p,b,p,c,p,c,c,p,b,p,c,p,p,b,c,p,b,p
	4-Type	p,c,b,c,t,b,c,b,t,p,c,b,c,p,c,b,t,c,p,b,t,c,b,b,t,c,b,p,c,b	t,p,c,p,b,t,p,b,t,c,p,ct,p,t,p,b,p,t,p,t,b,c,t, p,b,t,p,c,t
	5-Type	t,c,t,p,o,p,t,c,o,b,o,t,p,c,o,b,p,t,o,p,c,o,t,p,b,t,c, p,o,t,p,o,b,t,p,o,t,p,o,b	p,b,c,o,b,t,c,b,c,p,o,b,t,c,b,c,o,b,p,c,b,t,p, c,b,c,t,b,o,c,p,o,b,c,t
2	1-Type	p,p,p,p,p,p	p,p,p,p,p,p,p,p,p
	2-Type	p,c,c,p,c,p,c,p,c,c,c,c,p,c,p	c,p,p,c,p,p,c,p,p,c,p,c,p,p,c
	3-Type	b,c,p,c,b,c,p,b,c,b,c,p,c,b,p,b,c,p,c,b,b,c,p,b	p,c,b,p,p,c,p,b,p,c,b,b,p,c,p,c,p,b,p,c,b
	4-Type	t,c,b,p,b,c,c,p,t,b,c,b,t,c,p,c,p,b,t,t,b,c,p,c,b,b,c,p,t,b	c,t,p,b,p,t,t,c,b,p,t,p,c,t,b,t,p,b,c,c,p,t,b,t,p, p,t,b,c,p
	5-Type	t,c,b,o,o,p,b,c,c,t,p,o,b,c,b,o,t,c,o,p,c,b,p,t,t,o,b,o, c,p,c,b,b,b,o,o,p,t,b	c,t,p,o,o,b,p,t,t,c,b,p,t,p,o,c,t,o,b,t,p,b,c,c, p,o,t,b,t,p,p,t,o,b,c,p
3	3-Type	c,c,p,b,p,c,c,b,c,p,c,p,c,b,c,b,p,c,c,p,c,b,c,p,c,b,c,b	c,p,b,b,p,c,p,b,p,b,p,c,c,b,p,b,b,p,p,b,p,c,b, p,b,b,p,c,b,p,b,p,c,p,b
	4-Type	c,t,t,p,b,c,t,c,b,p,t,c,p,c,t,b,t,c,p,c,t,c,b,t	P,b,c,b,p,t,p,c,b,b,t,p,c,b,p,t,b,p,b,p,t,p,b,c

Abbreviations: (p: toy pig), (c: candy), (b: battery), (t: poker chip), (o: cottonball).

References

Alvarez, G. A., & Cavanagh, P. (2004). The capacity of visual short-term memory is set both by visual information load and by number of objects. *Psychological Science, 15*, 106–111.

Ansari, D., Dhital, B., & Soon, C. S. (2006). Parametric effects of numerical distance on the intraparietal sulcus during passive viewing of rapid numerosity changes. *Brain Research, 1067*, 181–188.

Barth, H., Kanwisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition, 86*, 201–221.

Barth, H., LaMont, K., Lipton, J., & Spelke, E. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences, 102*, 14116–14121.

Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin and Review, 8*, 698–707.

Cowan, N. (2001). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences, 24*, 87–185.

Dehaene, S. (1997). *The number sense*. Oxford University Press.

Erlick, D. E. (1964). Absolute judgments of discrete quantities randomly distributed over time. *Journal of Experimental Psychology, 67*, 475–482.

Feigenson, L., & Carey, S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568–584.

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object arrays. *Cognition, 97*, 295–313.

Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object-files vs. analog magnitudes. *Psychological Science, 13*, 150–156.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307–314.

Feron, J., Gentaz, E., & Streri, A. (2006). Evidence of amodal representation of small numbers across visuo-tactile modalities in 5-month-old infants. *Cognitive Development, 21*, 81–92.

Halberda, J., Sires, S. F., & Feigenson, L. (2006). Multiple spatially overlapped sets can be enumerated in parallel. *Psychological Science, 17*(7), 572–576.

Jordan, K. E., & Brannon, E. M. (2006). The multisensory representation of number in infancy. *Proceedings of the National Academy of Sciences of the United States of America, 103*, 3486–3489.

Karsh, R. (1969). Keeping track of sequential events: Multiple tallies and information rate. *Journal of Experimental Psychology, 84*, 339–342.

Kobayashi, T., Hiraki, K., & Hasegawa, T. (2005). Auditory-visual intermodal matching of small numerosities in 6-month-old infants. *Developmental Science, 8*, 409–419.

Lipton, J., & Spelke, E. (2003). Origins of number sense. *Psychological Science, 14*, 396–401.

Luck, S. J., & Vogel, E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature, 390*, 279–281.

Meck, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes, 8*, 226–243.

Monty, R. A., Taub, H. A., & Laughery, K. R. (1965). Keeping track of sequential events: Effects of rate, categories, and trial length. *Journal of Experimental Psychology, 69*(3), 224–229.

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature, 215*, 1519–1520.

Nieder, A., Diester, I., & Tudusciuc, O. (2006). Temporal and spatial enumeration processes in the primate parietal cortex. *Science, 313*, 1431–1435.

Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science, 297*, 1708–1711.

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human parietal cortex. *Neuron, 44*, 547–555.

Pinel, P., Piazza, M., LeBihan, D., & Dehaene, S. (2004). Distributed and overlapping representations of number, size, and luminance in parietal cortex during comparative judgments. *Neuron, 41*, 983–993.

Pylyshyn, Z. W., & Storm, R. W. (1988). Tracking multiple independent targets: Evidence for a parallel tracking mechanism. *Spatial Vision, 3*, 179–197.

Scholl, B. (2001). Objects and attention: The state of the art. *Cognition, 80*, 1–46.

Simon, T. J. (1997). Reconceptualizing the origins of number knowledge: A non-numerical account. *Cognitive Development, 12*, 349–372.

Uller, C., Huntley-Fenner, G., Carey, S., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive Development, 14*, 1–36.

Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*, 130–137.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15–B25.

Xu, Y., & Chun, M. (2006). Dissociable neural mechanisms supporting visual short-term memory for objects. *Nature, 440*, 91–95.

Yntema, D. B., & Mueser, G. E. (1960). Remembering the present states of a number of variables. *Journal of Experimental Psychology, 60*, 18–22.

Yntema, D. B., & Schulman, G. M. (1967). Response selection in keeping track of several things at once. *Acta Psychologica, 27*, 316–324.