PSYCHOLOGICAL SCIENCE

Research Report

Multiple Spatially Overlapping Sets Can Be Enumerated in Parallel

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ABSTRACT—***A system for nonverbally representing the approximate number of items in visual and auditory arrays has been documented in multiple species, including humans. Although many aspects of this approximate number system are well characterized, fundamental questions remain unanswered: How does attention select which items in a scene to enumerate, and how many enumerations can be computed simultaneously? Here we show that when presented an array containing different numbers of spatially overlapping dots of many colors, human adults can select and enumerate items on the basis of shared color and can enumerate approximately three color subsets from a single glance. This three-set limit converges with previously observed three-item limits of parallel attention and visual short-term memory. This suggests that participants can select a subset of items from a complex array as a single individual set, which then serves as the input to the approximate number system.***

Many species, including humans, can represent the approximate number of items in visual or auditory arrays without verbally counting (e.g., for reviews, see Dehaene, Dehaene-Lambertz, & Cohen, 1998, and Feigenson, Dehaene, & Spelke, 2004). Researchers have characterized various aspects of this enumeration ability, including its developmental progression (Lipton & Spelke, 2003; Siegler & Opfer, 2003) and its likely location in a region of the intraparietal sulcus in humans (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; but see Shuman & Kanwisher, 2004).

This *approximate number system* allows for the recognition of numerical quantities. But which items does it actually enumerate? A glance at any natural scene reveals the potentially limitless sets of items that might be enumerated (e.g., the trees, buildings, pedestrians, etc.). Yet at any given time, one is aware of the numerosity of only a few of these possible subsets; indeed, often one is unaware of numerosity altogether. This raises several questions regarding the nature of numerical processing.

First, what is the process by which some items in a scene are selected for enumeration while others are ignored? In previous research on the approximate number system, participants have been required to enumerate only a single set of items (e.g., all the dots on a screen). The role of attentive selection in forming number representations is therefore not understood.

Second, if enumeration of a subset is possible, can multiple subsets be enumerated at once? For example, could one look outside and simultaneously represent both the number of trees and the number of pedestrians without counting verbally? Although this question has never been tested explicitly, previous work found that rats treated stimuli with different perceptual properties (light flashes and auditory tones) as a single stimulus set, such that two flashes and two tones were treated as four events (Church & Meck, 1984).

To address both of these questions, we created a task in which participants saw briefly flashed arrays containing from 1 to 35 dots. Participants had to enumerate either all the items or just a subset defined by a common color. Items were completely spatially intermixed. On some trials, participants were informed in advance which subset they were to enumerate (probe-before trials). On other trials, participants were not told which subset to enumerate until after the array had been flashed (probe-after trials).

In Experiment 1, we used probe-before trials to investigate whether participants could attentively select which items to enumerate. We also compared performance on probe-before and probe-after trials to determine the number of subsets participants could enumerate in parallel from a single array. In Experiments 2 and 3, we confirmed that participants were responding

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to discrete number and not to continuous dimensions such as total area or circumference.

EXPERIMENT 1

Method

Ten naive adults with normal vision received course credit for participation. They viewed 450 trials on a Macintosh iMac computer (with a CRT monitor having a viewable area measuring 29.5 cm by 22.5 cm). Viewing distance was unconstrained, but averaged approximately 50 cm. The diameter of each dot subtended approximately 1° visual angle from a viewing distance of 50 cm.

On each trial, participants saw a 500-ms display containing 1 to 35 dots of one to six colors (red, blue, yellow, green, cyan, magenta; Fig. 1a illustrates an example trial). The number of dots in each color subset was randomly determined with the constraint that the target subset was smaller than at least one distractor subset on half of the trials. This made strategies such as "attend only the largest subset" ineffective. Dot position was randomly determined with the constraint that dots never overlapped. The 450 trials were presented in a single block in randomized order. A single color subset or the superset was probed on each trial. Participants typed their numerosity judgments on the numeric keypad.

Results and Discussion

Previous research shows that when the approximate number system is engaged, both the mean and the standard deviation of numerical estimates increase linearly as a function of the number of items to be enumerated, resulting in a constant coefficient of variance ranging from approximately .14 to .4 in human adults (Cordes, Gelman, & Gallistel, 2001; Whalen, Gallistel, & Gelman, 1999). We first asked whether participants' responses accorded with this pattern on probe-before trials containing dots of just a single color (one-color probe-before trials—e.g., a trial on which a probe telling participants to enumerate the blue dots is followed by an array containing 15 blue dots and no other dots). On such trials, no attentive selection of a subset was required. To have a sufficient number of trials to analyze both the mean and the standard deviation of responses, we combined trials from every participant in groups of three consecutive target numerosities (e.g., responses from trials with target numerosities of 7, 8, or 9 were combined and are shown at their mean, 8, in Fig. 2a). We considered only target numerosities 7 through 30 because we wanted to compare performance on one-color trials with performance on multiple-color trials, and on six-color trials there had to be at least 6 total dots and target numerosities above 30 could not occur (because displays were limited to a maximum of 35 dots). Figure 2a shows that both the mean and the standard deviation of participants' responses increased linearly, $t(6) = 24.1$, $p < .001$, $p_{\text{rep}} = .999$, $d = 19.65$, and $t(6) = 5.602$, $p < .001$, $p_{\text{rep}} = .988$, $d = 4.57$, respectively. The average coefficient of variance was .17. These results demonstrate that our task engaged the approximate number system.

Next, we asked whether selective attention operates prior to enumeration. If participants can attend to and enumerate just a subset of the array, then the performance signature on multiple-color probe-before trials (e.g., a trial on which a probe telling participants to enumerate the blue dots is followed by an array containing red, blue, and green dots) should be identical to the performance signature on one-color probe-before trials. Figure 2b shows that on multiple-color probe-before trials, the signature of the approximate number system was again observed. Both the mean and the standard deviation of responses increased linearly, $t(6) = 17.21$, $p < .001$, $p_{\text{rep}} = .999$, $d = 14.05$, and $t(6) = 15.702$, $p < .001$, $p_{\text{rep}} = .999$, $d = 12.82$, respectively, and the average coefficient of variance was .21. Indeed, the percentage error in participants' estimates did not increase

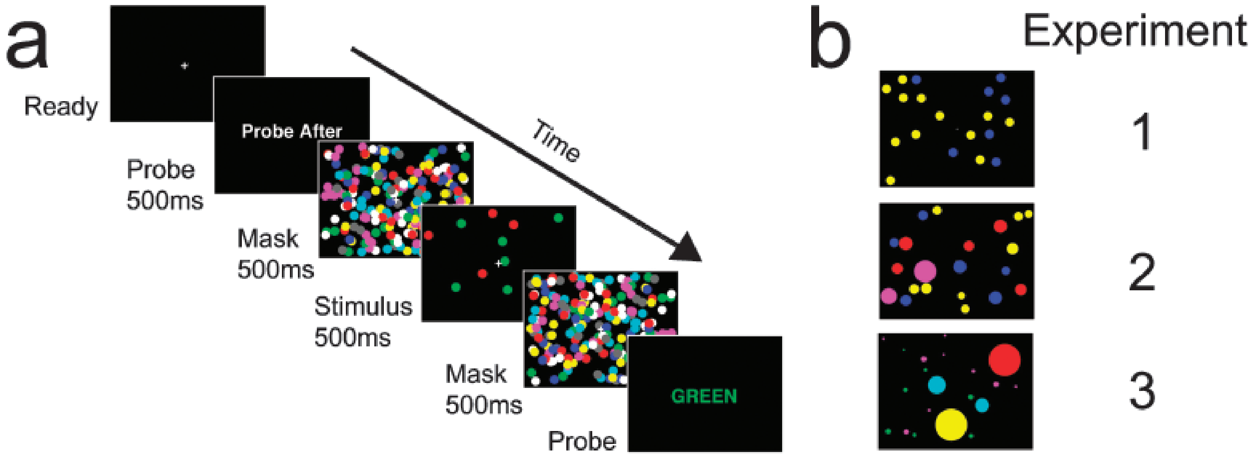

Fig. 1. Illustration of a two-color probe-after trial from Experiment 1 (a) and sample stimulus arrays from Experiments 1 through 3 drawn to scale (b). Example trials can be viewed on the Web at http://www.psy.jhu.edu/~halberda/demos.html.

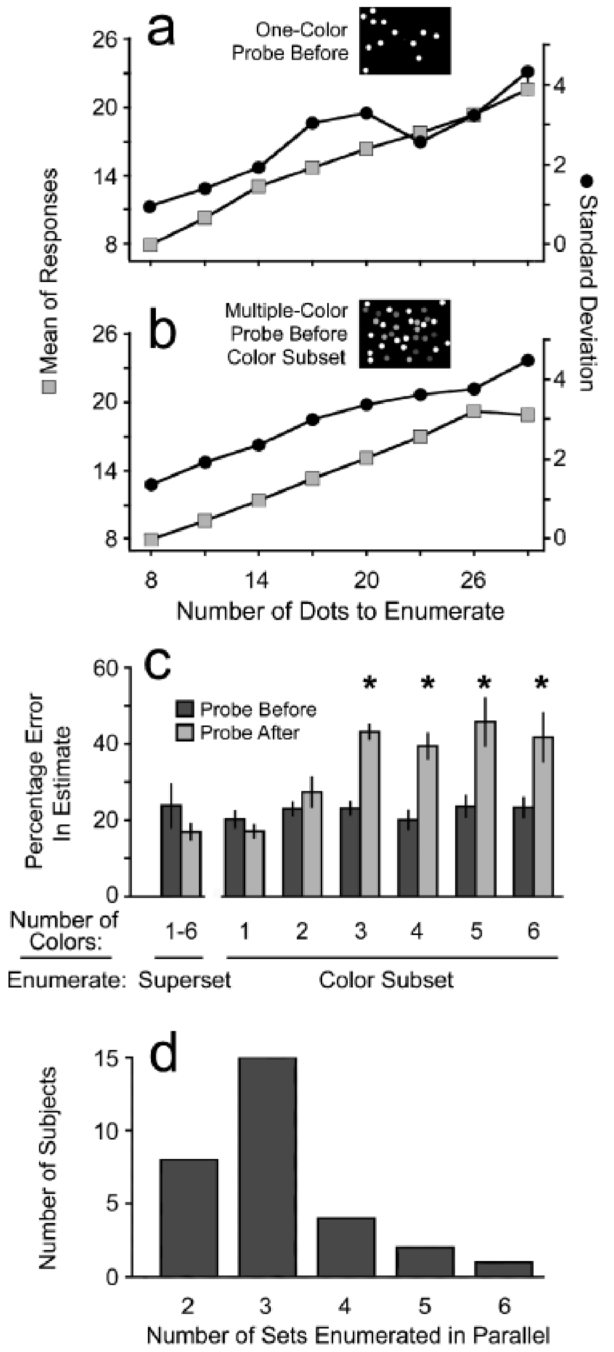

Fig. 2. Experimental results. The top two graphs present mean response and standard deviation as a function of the number of dots to be enumerated in Experiment 1. Results are shown separately for (a) one-color probe-before trials and (b) multiple-color probe-before trials. The bar graph in (c) presents percentage error for probe-before and probe-after trials for participants in Experiment 1 who succeeded at enumerating three sets in parallel. For color-subset trials, on which only a single subset was enumerated, percentage error is graphed as a function of the number of colors in the array; for superset trials, on which participants enumerated all the dots in the display irrespective of color, percentage error did not vary as a function of number of colors in the array. Error bars show standard errors. Asterisks indicate conditions in which there was a significant difference in percentage error between probe-before and probe-after trials ($p < .05$). The histogram in (d) shows the number of sets participants could enumerate in parallel in Experiments 1 through 3 ($M = 3.1$).

as the number of color subsets in the display increased, as shown by the nonsignificant linear regression of percentage error on the number of color subsets in the stimulus (1–6), $t(4) = 1.031$, $p = .361$ (e.g., in Fig. 2c, see the flat error rates on probe-before trials for a subset of the participants involved in this regression).

Finally, we asked whether adults can enumerate multiple subsets of dots in parallel. Probe-before and probe-after trials formed yoked pairs: For each probe-before trial, there was a matched probe-after trial. The particular color for each color subset and the position of each dot varied within yoked pairs to disable any perceptual learning across trials. If participants could enumerate all of the color subsets in the displays, then error on probe-after trials would not differ from error on probe-before trials, despite the fact that on probe-after trials participants did not know which color subset to enumerate until after the display had disappeared. Representing only some of the color subsets would result in participants sometimes not knowing the correct answer to the probe. This would lead to a difference in performance between probe-before and probe-after trials. For each participant, planned *t* tests compared error on probe-before and probe-after trials. The number of colors at which error on probe-after trials became significantly higher than that on probe-before trials indicated the number of sets the participant could encode and recall from a single display.

We found that most participants encoded three sets from a single display ($M = 2.6$, range = 2–4). Figure 2c displays the error pattern for the participants in Experiment 1 who enumerated three sets in parallel. Percentage error on probe-before trials did not increase as the number of color subsets in the stimulus increased, which shows that attention was efficient at selecting which subset to enumerate. In contrast, percentage error on probe-after trials increased abruptly when the stimulus included three or more color subsets. We took this categorical increase in percentage error on probe-after trials to indicate the number of sets each participant could enumerate in parallel. For example, for participants who enumerated four sets in parallel, the results were the same as those displayed in Figure 2c with the exception that the first significant difference in performance occurred on trials containing four color subsets in the stimulus, rather than three.

On 13% of trials, participants were prompted to report the total number of dots in the display irrespective of color (superset trials). Results indicated that participants always selected the superset of all dots for enumeration on probe-after trials, irrespective of the number of colors in the display. Percentage error did not differ between probe-before and probe-after superset trials for any participant, regardless of the number of colors shown (all ps $> .05$). The superset of all dots appears to have been represented as a single set, rather than as the sum of subsets, for two reasons. First, participants succeeded at enumerating the superset on probe-after trials containing up to six color subsets. This was beyond the number of separate color subsets that any participant could enumerate. Second, on superset probe-after trials, the number of colors in the array did not have a significant effect on reaction time as measured by a repeated measures analysis of variance, $F(5, 45) = 1.150$, $p = .348$. The fact that participants had a constant reaction time regardless of how many colors were in the array suggests that additional colors added no time to the computation of numerosity. Thus, for participants who encoded three sets in parallel, these sets were the superset of all dots and two color subsets (Fig. 2c).

The three-set limit on enumeration observed in the present study converges with the three-item limits that have been observed in studies of object-based attention (Pylyshyn & Storm, 1988; Scholl, 2001) and visual short-term memory (Alvarez & Cavanagh, 2004; Luck & Vogel, 1998). The agreement between the set-based limit we observed in the present study and previously observed item-based limits suggests that each set of dots in Experiment 1 was attended and stored as an individual, despite the fact that the individual dots within each set remained available for enumeration by the approximate number system. Previous work has shown that adults can attend at most three or four individuals, when individuals are roughly bounded, coherent objects (Scholl, 2001) or spatially bounded groups (e.g., flocks of birds; vanMarle & Scholl, 2003; Wynn, Bloom, & Chiang, 2002). The present work shows that even a nonobject entity that does not occupy a bounded spatial location, such as “the set of red dots” spatially intermixed with other dots, can function as an individual for attention and short-term memory, and that approximate number can be stored as a feature of such an individual.

EXPERIMENTS 2 AND 3

To ensure that participants’ responses were based on number rather than on any continuous dimension correlated with number, we replicated Experiment 1 controlling for area (Experiment 2) and contour length (Experiment 3), two dimensions to which infants have shown sensitivity given similar visual arrays (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002).

Method

Twenty new naive adults with normal vision received course credit for participation ($n = 10$ for each experiment). Methods were identical to those of Experiment 1 except that on every trial, each color subset was constrained to have either a total cumulative area of 26,214 pixels (Experiment 2) or a total cumulative perimeter (i.e., cumulative circumference) of 667 pixels (Experiment 3; Fig. 1b). Thus, total area or circumference no longer covaried with number. All items were circular, but of varying sizes. The size of each dot within a color subset was randomly selected from a range with lower and upper bounds of 50% and 150% of the average size for that set. Thus, although total area or circumference was constant for every set, individual dot sizes varied, discouraging the use of dot size as a cue to numerosity.

Results and Discussion

The pattern of results was the same as in Experiment 1. On one-color probe-before trials, both the mean and the standard deviation of responses increased linearly: $t(6) = 13.67$, $p < .001$, $p_{\text{rep}} = .999$, $d = 11.16$, and $t(6) = 2.50$, $p < .05$, $p_{\text{rep}} = .921$, $d = 2.04$, respectively, for Experiment 2; $t(6) = 12.708$, $p < .001$, $p_{\text{rep}} = .999$, $d = 10.38$, and $t(6) = 9.224$, $p < .001$, $p_{\text{rep}} = .997$, $d = 7.53$, respectively, for Experiment 3. The coefficient of variance was .34 in both experiments. A similar pattern was observed on multiple-color probe-before trials: $t(6) = 23.22$, $p < .001$, $p_{\text{rep}} = .999$, $d = 18.96$, and $t(6) = 10.633$, $p < .001$, $p_{\text{rep}} = .998$, $d = 8.68$, respectively, for Experiment 2; $t(6) = 6.125$, $p < .001$, $p_{\text{rep}} = .991$, $d = 5.00$, and $t(6) = 7.686$, $p < .001$, $p_{\text{rep}} = .995$, $d = 6.28$, respectively, for Experiment 3. The coefficient of variance was .31 in Experiment 2 and .39 in Experiment 3. These results indicate that participants were able to select a subset of items that shared a common color and enumerate these items even when total surface area or circumference did not covary with number. The coefficient of variance observed in Experiments 2 and 3, although within the acceptable range for human adults, was higher than that observed in Experiment 1. This difference may indicate that participants in Experiment 1 were relying on multiple cues (e.g., number and area), whereas participants in Experiments 2 and 3 could rely only on number. New experiments in our lab are testing this hypothesis.

As in Experiment 1, we compared each participant’s performance on probe-before versus probe-after trials in order to determine the number of sets that each participant could enumerate in parallel. We again found that most participants enumerated approximately three sets from a single display (Experiment 2: $M = 3.4$, range = 2–6; Experiment 3: $M = 3.2$, range = 2–5). Given the similar patterns of performance across our three experiments, the data can be collapsed to show the number of subsets participants were able to enumerate. Figure 2d shows that the mode was three.

GENERAL DISCUSSION

Our results specify at least two critical new aspects of numerical processing. First, and most surprisingly, our results demonstrate that people are able to enumerate multiple sets of items in parallel. The three-set limit observed converges with the three-item limits of visual attention (Pylyshyn & Storm, 1988; Scholl, 2001) and visual short-term memory (Alvarez & Cavanagh, 2004; Luck & Vogel, 1988). Our results suggest that about three individual sets can be stored in short-term memory, and that approximate number can be stored as a feature of each set.

Second, our results suggest that attentive selection is necessary prior to enumeration. Participants had no difficulty enumerating spatially intermixed subsets within a scene when these subsets were selectable on the basis of a common early visual feature. It remains to be tested whether other early visual features, such as size or orientation, can also serve as the basis for the selection of to-be-enumerated items.

Considered from a broad perspective, our results illustrate the degree to which items in a visual scene can be represented hierarchically. For example, a single display of 12 blue, 5 red, 1 green, 3 magenta, and 7 yellow dots can be represented as one array, five groups, or 28 separate dots. At one level, "the set of blue dots" may be selected and stored as a single individual in visual short-term memory. At another level, "the set of blue dots" may be treated as 12 distinct items, available for enumeration by the approximate number system. This distinction highlights a hierarchical coding of "set" and "individual" that is important for all mathematical concepts. Furthermore, our results indicate that the notion of a set may operate prior to enumeration by the approximate number system in adults, converging with evidence that "set" may be an important concept early in child development (Feigenson & Halberda, 2004).

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