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Infants Extract Frequency Distributions from Variable Approximate Numerical Information

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Previous research shows that infants represent approximate number: After habituation to a constant numerosity (e.g., eight dots), 6-month-old infants dishabituate to a novel numerosity (e.g., 16 dots). However, numerical information in the real world is far more variable and rarely offers repeated presentations of a single quantity. Instead, we often encounter quantities in the form of distributions around a central tendency. It remains unknown whether infants can represent frequency distributions from this type of distributed numerical input. Here, we asked whether 6-month-old infants can represent distributions of large approximate numerosities. In two experiments, we first familiarized infants to sequences of dot collections with varying numerosities. For half the infants, the sequence contained a unimodal frequency distribution, with numerosities centered around a single mean, and for the other half, it contained a bimodal frequency distribution of numerosities with two numerical peaks. We then tested infants with alternating or constant numerosities. Infants who had been familiarized to a unimodal distribution looked longer at alternating numerosities than constant numerosities (experiments 1 and 2), whereas infants who had been familiarized to a bimodal distribution looked longer at constant numerosities (Exp. 2). These findings suggest that infants can spontaneously extract frequency distributions from distributed numerical input.

From birth, infants spontaneously extract numerical information from visual and auditory stimuli. For example, newborns look longer at visual arrays that numerically match an auditory sequence than at arrays that numerically mismatch (Izard, Sann, Spelke, & Streri, 2009). By 6 months of age, infants habituated to one numerical

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quantity dishabituate when presented with a new quantity, both for visual and auditory stimuli (Lipton & Spelke, 2003; Xu & Spelke, 2000). Furthermore, infants can mentally manipulate numerosities. For example, 9-month-old infants look longer at the incorrect result of visually presented addition problems (5 objects + 5 objects = 5 objects) than at the correct result (5 + 5 = 10) (McCrink & Wynn, 2004).

The numerical representations infants rely on in these tasks are noisy and inexact, as shown by the ratio-dependent nature of their performance. At 6 months of age, infants can reliably discriminate quantities that differ by a 1:2 ratio (e.g., if habituated to eight dots they will dishabituate to 16 dots, and vice versa), but not quantities that differ by a 2:3 ratio (e.g., if habituated to eight dots, they fail to dishabituate to 12, and vice versa) (Feigenson, 2011; Lipton & Spelke, 2003; Xu & Spelke, 2000). The noisiness of these approximate number representations decreases rapidly over development; by 9 months, infants successfully discriminate quantities that differ by a 2:3 ratio (Lipton & Spelke, 2003; Xu & Arriaga, 2007), and their precision continues to improve throughout childhood (Halberda & Feigenson, 2008). However, even though approximate number representations grow more precise with age, they never come to support exact number representation—for example, neither infants nor adults can use approximate number representations to tell apart arrays of 20 vs. 21 dots. That degree of precision requires a different system of numerical representation—one that represents exact integers, and that is acquired (typically in early childhood) via language and learning of the counting routine (Carey, 2009; Wynn, 1992).

Much of what we know about the nature of approximate number representations early in development has been revealed by habituation studies in which one numerosity is presented repeatedly, until infants' looking at that numerosity has significantly diminished. Although non-numerical aspects of the habituation stimuli may vary from trial to trial (e.g., spatial arrangement, item size, cumulative extent, item spacing), numerosity remains constant. In the subsequent test phase, infants' looking to the habituated numerosity is compared to their looking to a novel numerosity. Successful numerical discrimination is inferred from significantly longer looking at the numerically novel stimulus. What we can conclude from studies using this method is that infants can form a representation of approximate numerosity from repeated exposures to a single value, and detect when this repeated numerosity changes (Brannon, Abbott, & Lutz, 2004; Cordes & Brannon, 2009; Jordan, Suanda, & Brannon, 2008; Libertus, Brannon, & Woldorff, 2011; Lipton & Spelke, 2003, 2004; Xu, 2003; Xu & Arriaga, 2007; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005; Zosh, Halberda, & Feigenson, 2011).

Even though in this experimental paradigm infants see the same numerosity repeatedly, current theories of approximate number representation suggest that repeated presentations of an external stimulus containing, for example, 16 dots, will trigger a range of mental activations that form a distribution with some mean (at approximately 16) and a standard deviation (representational noisiness) that varies across observers of different ages, and even across individuals of the same age (Halberda & Feigenson, 2008; Halberda, Ly, Willmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008). One possibility is that a noisy distribution is represented each time 16 items are experienced in the world (Halberda, 2016; Halberda & Odic, 2014; Odic, Im, Eisinger, Ly, & Halberda, 2015). Alternatively, under a sampling assumption, each time 16 items are encountered, a single discrete sample might be drawn from this mental distribution. Because of the variability of the system, most samples will be only slightly greater than or smaller than 16, fewer samples will be more distant from 16, and yet fewer samples will be quite different than 16 (and different enough to be confusable with a different distribution, centered on a relatively distant stimulus value, like 24). Under either the distribution or the sampling model, approximate number representations are "noisy" and distributed, with increasing noise—or less certainty (Halberda & Odic, 2014)—as numerosities increase.

Although much has been learned about early quantitative competence from studies that tested infants' ability to represent a constant numerosity, real-world numerical experience is far more variable. Rather than repeated exposures to a single quantity, we often are confronted with quantities that exhibit variability but that cluster around some central tendency. For example, although every morning an infant might be offered a handful of roughly 20 Cheerios on her high chair tray, the exact number of Cheerios given likely varies from day to day. Indeed, although the mean number of Cheerios across these repeated presentations might be 20 (e.g., the infant was handed 15 Cheerios one day, 25 the next, then 18, 22, etc.), a presentation of precisely 20 Cheerios might never have occurred. What do infants represent, given this type of variable numerical experience? Despite much research characterizing early numerical abilities, no evidence currently bears on the question of whether and how infants represent numerical information from numerically variable experiences. Given noisy input in which numerosity varies, how do infants decide whether they are experiencing a single numerical distribution (with some mean and some standard deviation), and when they are instead experiencing several different distributions? To return to the Cheerios example, imagine that the number of Cheerios placed on the infant's tray varies. How do infants know whether they are always seeing presentations of "about 20" Cheerios, with a large amount of variance (such that sometimes 30 Cheerios are given, and sometimes just 10, but with a peak at 20), vs. seeing, for example, a bimodal distribution resulting from sometimes receiving 10 Cheerios (with some variance around 10) and sometimes receiving 40 (with some variance around 40)? Can infants use numerical variability across experiences to determine how many distributions are present?

While there is no research on whether infants can represent numerical information from noisy distributions, infants' abilities in non-numerical domains offer some insights and highlight the very general nature of the problem infants face. Parsing continuous stimuli into discrete representations is a challenge that learners must solve across a wide range of domains. One well-studied case is phoneme discrimination. Infants hear phonemes that vary continuously along physical dimensions such as voice onset time. From this continuous input, infants must "find" phonemic categories. Maye, Werker, and Gerken (2002) suggested that infants can extract frequency distributions to solve this problem. They familiarized 6- and 8-month-old infants from English-speaking families to a continuum between voiced and voiceless unaspirated alveolar stops [d] and [t], which are not treated as distinct phonemes in these infants' native language. Infants in one condition were familiarized to a unimodal distribution of stimuli along the [d]–[t] continuum, with a single peak around the continuum's middle (i.e., stimuli from the middle of the continuum were played most frequently). Infants in the other condition were familiarized to a bimodal distribution with two peaks near the continuum's ends (i.e., stimuli from near the ends of the continuum were played most frequently). Subsequently, all infants were tested with alternating stops that were drawn from the very ends of the continuum, and nonalternating stops that were drawn from a region closer to the middle. Importantly, infants in both familiarization conditions had had equal exposure to these specific test stimuli during familiarization. Yet, infants who had been familiarized to the bimodal distribution attended longer to trials containing nonalternating stops (i.e., these infants had apparently formed representations of two phonemic categories during familiarization, and the nonalternating stops in the middle were perceived as not belonging to either of these categories). In contrast, infants familiarized to a unimodal distribution showed no significant test preference—presumably because they experienced the Unimodal Familiarization stimuli as containing just a single quite broad distribution or phonemic category and subsequently perceived all of the test stimuli, whether alternating or nonalternating, as belonging to that category. These results suggest that infants are sensitive to frequency distributions and can use these distributions as a basis for identifying meaningful categories, at least in the domain of phonology. This work inspired us to ask whether this type of distribution-based learning occurs in the domain of large approximate number.

In this study, we asked whether experiencing variable numerical input would lead infants to represent and discriminate frequency distributions of numerosities. As a first small step, in Experiment 1 we asked whether infants could discriminate between "unimodal" and "bimodal" distributions involving only a single or two repeated values. This was a prudent step as it is the middle ground between the familiarization methods of Maye et al. (2002) and the traditional numerical discrimination studies which use constant values. In Experiment 1, infants either were familiarized to a single numerical value (24 dots) presented repeatedly across a series of trials, or to alternating presentations of two different numerical values (16 and 36 dots). Then all infants were tested with both alternating and constant numerical sequences. We predicted that if infants can appreciate the unimodal vs. bimodal nature of the familiarization stimuli, then their preferences for the test stimuli would be significantly affected by their familiarization condition. Note, however, that if experienced collections give rise to "noisy" distributions in the mind (Halberda, 2016; Halberda & Odic, 2014; Odic et al., 2015), then Experiment 1 also tests distributions; that is, distributions with a standard deviation specified by the internal precision of our numerical representations.

In Experiment 2, we increased the complexity of the familiarization stimuli to better reflect dynamic real-world experience. All infants were familiarized to sequences containing varying numerosities over the same number range. Infants either were familiarized to varying numerosities with a single numerical peak (i.e., a unimodal distribution), or to varying numerosities with two numerical peaks (i.e., a bimodal distribution). All infants were then tested with sequences that alternated in number and sequences that remained constant in number. We predicted that if infants can parse frequency distributions from continuously varying familiarization input, they would prefer test arrays containing a novel number of distributions. That is, infants familiarized to a unimodal distribution should prefer the numerically alternating test trials, and infants familiarized to a bimodal distribution should prefer the numerically constant test trials.

EXPERIMENT 1

Method

Participants

Thirty-two healthy, full-term 6-month-old infants (mean age = 179.84 days, $SD = 9.68$ days; 21 females) participated. Data from 12 additional infants were excluded for fussiness ($n = 5$), parental interference ($n = 4$), or experimenter error ($n = 3$). Data from one additional infant were excluded because her looking times during familiarization were more than two standard deviations above the group average. Infants were recruited by mail and by telephone, as approved by the local Institutional Review Board, and all parents provided written informed consent prior to their child's participation. All children received a small gift (e.g., a small toy, a T-shirt, or a book) to thank them for their participation.

Design and materials

Half of the infants were randomly assigned to the Unimodal Familiarization condition, and the other half were assigned to the Bimodal Familiarization condition. We used a familiarization procedure rather than an infant-controlled habituation procedure because we wished to use the same procedure for both experiments 1 and 2, and in Experiment 2, it was critical that we would be able to precisely control the number of times infants saw each numerosity (Maye et al., 2002). As such, all infants received eight familiarization trials, where each trial consisted of a cycling sequence of briefly presented arrays.

In the Unimodal Familiarization condition the presented numerosity remained constant across these arrays—infants were familiarized only to arrays containing 24 circles (Figure 1), but non-numerical aspects such as cumulative surface area and density varied. In the Bimodal Familiarization condition the presented numerosity alternated between 16 and 36 circles, and non-numerical aspects of the arrays also varied.

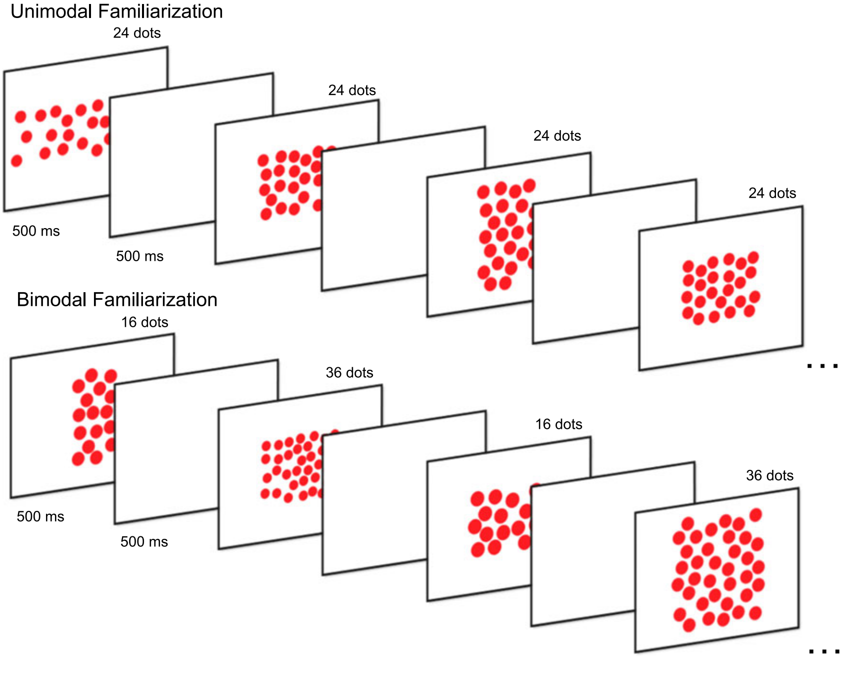

Figure 1 Schematic of sample familiarization stimuli in the Unimodal Familiarization condition (top) and the Bimodal Familiarization condition (bottom).

Familiarization was followed by six test trials that were identical across the two familiarization conditions. Three of these were Alternating Number test trials containing novel arrays of 16 and 36 circles shown in alternation, and three were Constant Number test trials containing novel arrays of 24 circles (Figure 2). The Alternating Number and Constant Number test trials were shown in alternation.

Stimuli consisted of images of 16, 24, or 36 red circles randomly arranged on a white background. During familiarization, half of the images contained circles with a 1 cm radius; for these images, the cumulative surface area of the circles increased with increasing number. The other half of the familiarization images contained circles with radii ranging from 1.09 to 0.73 cm; for these images, the cumulative surface area of the circles was equated across all of the presented numerosities. The test stimuli were circles whose radii ranged from 0.44 to 1.59 cm; for these images, the cumulative perimeter of the circles was equated across all of the presented numerosities. Orthogonally, in both familiarization and test, half of all images were equated for density; in the other half density increased with increasing number of circles. Each image was only used once during the testing session.

We would like to highlight one technical aspect of our stimulus selection. The numerical values in our experimental design were chosen so that we could test infants' detection of a change in the unimodal/bimodal *distribution* of stimulus numerosities, rather than their detection of a simple change in number. Previous studies show that 6-month-old infants detect a 1:2-ratio change (e.g., 16 to 32 dots) but not a 2:3-ratio change (e.g., 16 to 24 dots; Xu et al., 2005). Therefore, infants in the Unimodal Familiarization condition who were familiarized to 24 dots should not show a preference for the Alternating Number test trials purely on the basis of noticing a change from the

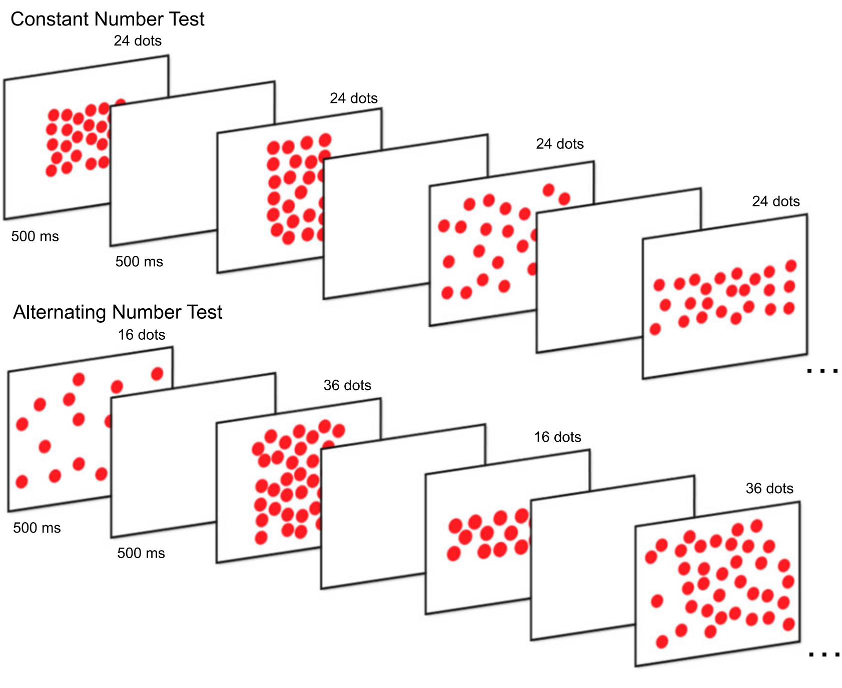

Figure 2 Schematic of sample test stimuli in the Constant Number test (top) and Alternating Number test (bottom).

familiarization numerosity (24), because neither 24:16 nor 24:36 instantiates a discriminable ratio for infants of this age. In contrast, the ratio between the two numerosities in the Alternating Number test trials was chosen to be discriminable to infants of this age (16:36). Hence, our design tested whether infants detected a change in the frequency distributions of the numerosities in familiarization vs. the distribution at test.

Procedure

Infants sat in a high chair or on a parent's lap in a dimly lit room, approximately 60 cm from a computer screen that was surrounded by a dark curtain. For infants seated in the high chair, parents sat approximately 60 cm behind infants and were asked not to interact throughout the experimental session. For infants seated on a parent's lap, parents were instructed to keep infants facing the screen but otherwise to avoid interacting throughout the experimental session. The experimenter controlled the study from behind the curtain and was not visible to infants during the experiment. A concealed video camera below the computer screen recorded infants' looking behavior for later offline coding. Classical music was played quietly in the background to create a calming testing environment.

Each trial was preceded by a colorful visual attractor (a spinning pinwheel) accompanied by a drumbeat to attract infants' attention to the screen. The experimenter manually initiated each trial as soon as infants looked at the screen. Each of the eight familiarization trials consisted of 16 images; each image was presented for 500 ms, followed by 500 ms of a blank screen (Figure 1). We chose a 500-ms stimulus presentation based on previous studies using a similar method (Libertus & Brannon, 2010; Libertus, Starr, & Brannon, 2014; Starr, Libertus, & Brannon, 2013a,b). Once started, each familiarization trial proceeded regardless of the direction of infants' gaze. Following the eight familiarization trials, all infants were shown the same six test trials, that is, three Alternating Number and three Constant Number trials, in alternation. Which of these appeared first was counterbalanced across infants. Each test trial consisted of 32 images; each image was presented for 500 ms, followed by 500 ms of a blank screen (Figure 2). As in the familiarization trials, the presentation of each test trial was initiated as soon as infants looked at the screen and proceeded regardless of the direction of infants' gaze throughout the trial.

Data analysis

An experienced observer coded infants' looking using a custom-made coding program written in RealBasic (Libertus, 2008). A second observer coded 25% of all participants' testing sessions, and the reliability between the two observers was high ($r = .98$). We analyzed the proportion of time infants spent looking at the screen out of the total duration of each trial. We then calculated separate averages for Familiarization, Alternating Number test, and Constant Number test trials.

Results

During familiarization, infants looked for an average of 69.08% ($SD = 13.09$, range: 43.44–92.97%) of the total time the numerical stimuli were visible. In general, infants' looking declined over the course of familiarization. Infants looked for an average of

80.1% (*SD* = 17.49) of the time on the first familiarization trial, and 60.5% (*SD* = 18.74) of the time on the last familiarization trial. Twenty-four of 32 infants looked longer on the first familiarization trial than the last. No significant difference in the percentage of looking throughout Familiarization was found between infants in the Unimodal (70.21%, *SD* = 14.82) and the Bimodal Familiarization (68.02%, *SD* = 11.64) conditions, $t(29) = 0.46$, $p = .65$, Cohen's $d = 0.16$.

We next asked whether infants' familiarization experience affected their later preference for arrays that alternated in numerosity vs. those that were constant in numerosity. We found that of the total amount of time infants looked at the stimuli during the test trials, infants in the Unimodal Familiarization condition spent an average of 51.52% (*SD* = 14.03) looking during the Alternating Number test trials and 43.61% (*SD* = 15.03) looking at the Constant Number test trials (see Figure 3). Infants in the Bimodal Familiarization condition spent an average of 49.23% (*SD* = 20.42) looking during the Alternating Number test trials and 50.62% (*SD* = 16.12 looking at the Constant Number test trials. A mixed-design ANOVA with Familiarization Condition (Unimodal or Bimodal Familiarization) as between-subjects factor and Test Trial Type (Alternating Number or Constant Number) as within-subjects factor yielded no significant main effects, but did reveal a significant interaction between Familiarization Condition and Test Trial Type, $F(1, 30) = 4.55$, $p < .05$, $\eta^2 = 0.13$. Infants in the Unimodal Familiarization condition showed a significant looking preference (for the Alternating Number trials) at test, $t(15) = 2.94$, $p = .01$, $d = 0.73$, whereas infants in the Bimodal Familiarization condition showed no significant preference, $t(15) = -0.41$, $p = .69$, $d = 0.10$.

Discussion

In Experiment 1, we found that 6-month-old infants who had been familiarized to a unimodal distribution of numerosities subsequently looked longer at test trials in which

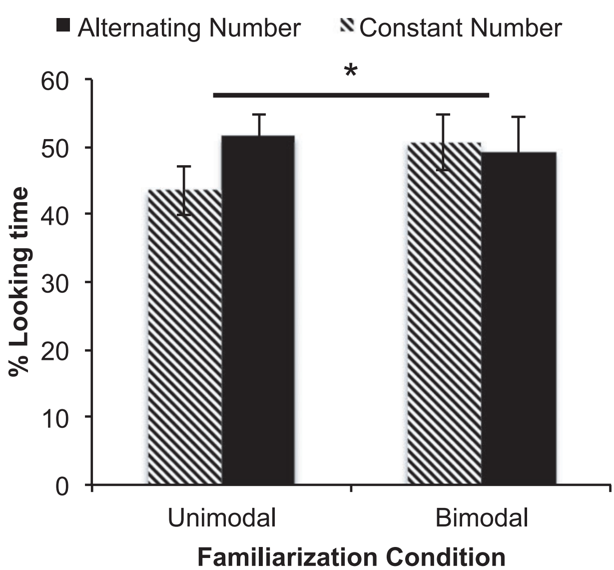

Figure 3 Average percent looking time for Alternating Number and Constant Number test trials for the two familiarizations condition in Experiment 1. Error bars reflect standard error of the mean.

two different numerosities alternated, compared to test trials in which the same numerosity was repeated. Infants appeared to detect a change from a unimodal numerical distribution (constant presentations of 24 items) to a bimodal alternating presentation of 16 and 36 items. This finding likely reflects infants' detection of a change in the distributions of numerosities across familiarization and test—and is not likely to be the result of infants simply detecting a change in numerosity from familiarization to test—because the ratio between the familiarization and test numerosities was designed to be below the threshold of discriminability for infants of this age.

Unexpectedly, we found that infants who had been familiarized to a bimodal distribution (alternating presentations of 16 and 36 items) subsequently did not show any preference at test to look at a unimodal distribution. One possible reason for this is that the Alternating Number test trials may have been inherently more interesting than the Constant Number Test trials due to their greater numerical variability. Previous research comparing infants' preference for numerically alternating image streams similar to those in our Alternating Test trials vs. numerically constant image streams similar to those in our Constant Number Test trials showed that—in the absence of familiarization to different frequency distributions—infants prefer the numerically alternating streams if they are able to discriminate the change in number (e.g., 1:2 ratio at 6 months of age; Libertus & Brannon, 2010). Thus, it is possible that this inherent bias for numerical variability counteracted the increase in looking to the novel Constant Number Test trials in the current experiment, effectively leading to no difference in looking between the familiar Alternating and novel Constant Number Test trials.

In Experiment 2, we increased the variability of both the Unimodal Familiarization and Bimodal Familiarization stimuli. In Experiment 2, we familiarized all infants to sequences of varying numerosities. One group of infants was familiarized to a sequence in which numerosity varied between 10 and 58, but in which there was a single distributional peak at 24 (although 24 itself was never seen). The other group of infants also was familiarized to a sequence in which numerosity varied between 10 and 58, but in which the distribution contained two numerical peaks at 12 and 48 (Figure 4). If infants represent these distributions, their looking preferences postfamiliarization should be differentially affected. Specifically, we predicted that infants familiarized to the unimodal distribution of numerosities would later prefer test sequences containing two alternating numerosities. In contrast, we predicted that infants familiarized to the bimodal distribution of numerosities would later prefer test sequences containing a single constant numerosity.

EXPERIMENT 2

Method

Participants

Thirty-two 6-month-old infants (mean age = 181.44 days, *SD* = 9.35 days; 18 females) who had not previously participated in Experiment 1 participated in Experiment 2. Data from thirteen additional infants were excluded due to fussiness ($n = 7$), parental interference ($n = 4$), prematurity ($n = 1$), or equipment failure ($n = 1$). Data from one additional infant were excluded because of looking times during

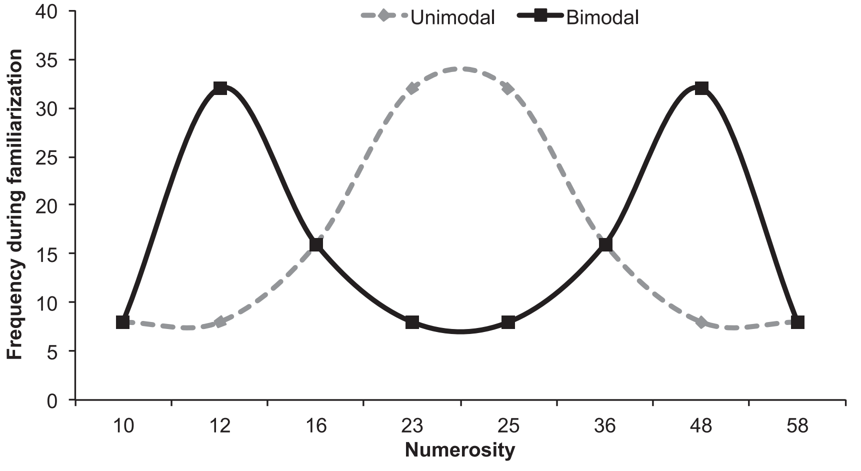

Figure 4 Frequency distributions of numerosities in the Unimodal Familiarization condition (gray) and the Bimodal Familiarization condition (black).

familiarization that were more than two standard deviations above the group average. As in Experiment 1, infants were recruited by mail and by telephone, as approved by the local Institutional Review Board, and all parents provided written informed consent prior to their child's participation. All children received a small gift (e.g., a small toy, a T-shirt, or a book) to thank them for their participation.

Materials

Stimuli consisted of images of 10, 12, 16, 23, 25, 36, 48, or 58 red circles randomly arranged on a white background. During familiarization, half of the images contained circles with a 1 cm radius; for these images, the cumulative surface area of the circles increased with increasing number. The other half of the familiarization images contained circles with radii ranging from 0.57 to 1.38 cm; for these images, the cumulative surface area of the circles was equated across all of the presented numerosities. The test stimuli were identical to Experiment 1. As in Experiment 1, each image was presented only once during the experimental session.

As in Experiment 1, all infants saw eight familiarization trials, with each familiarization trial consisting of a series of 16 briefly presented numerosities. Each numerosity was shown for 500 ms, followed by a 500 ms blank interval, followed by the next numerosity. For infants in the Unimodal Familiarization condition, each familiarization trial was comprised of a sequential stream that contained one image with 10, 12, 48, and 58 circles, respectively, two images with 16 and 36 circles, respectively, and four images with 23 and 25 circles, respectively (Figure 4). Thus, the mean of this Unimodal Familiarization distribution was 24. For infants in the Bimodal Familiarization condition, each familiarization trial contained one image with 10, 23, 25, and 58 circles respectively, two images with 16 and 36 circles, and four images with 12 and 48 circles, respectively. The modes of the Bimodal Familiarization distribution were 12 and 48. Within each familiarization trial, the order in which the different numerosities was shown was random.

The test trials were identical to those in Experiment 1. Infants saw three Alternating Number test trials in which images of 16 and 36 circles were shown in alternation, and three Constant Number test trials in which images of 24 circles were shown. Alternating Number and Constant Number test trials were shown in alternation. Note that there were no images with exactly 24 circles in either of the familiarization conditions and that infants in both familiarization conditions had equal exposure to images containing 16 and 36 circles. Thus, any differential responses during the test trials cannot be attributed to differences in exposure to these numerical values during familiarization.

Data analysis

An experienced observer coded infants' looking using a custom-made coding program written in RealBasic (Libertus, 2008). A second observer coded 25% of all participants' testing sessions, and the reliability between the two observers was high ($r = .97$).

Results

During familiarization, infants looked for an average of 64.89% ($SD = 13.65$, range: 41.32–89.46%) of the total time that the numerical stimuli were visible. As in Experiment 1, infants' looking declined over the course of familiarization. Infants looked for an average of 77.72% ($SD = 17.48$) of the time on the first familiarization trial, and 54.56% ($SD = 23.76$) of the time on the last familiarization trial. Twenty-seven of 32 infants looked longer on the first familiarization trial than the last. No significant difference in the percentage of looking throughout Familiarization was found between infants in the Unimodal (63.00%, $SD = 11.62$) and the Bimodal Familiarization (66.77%, $SD = 15.56$) conditions, $t(30) = -0.78$, $p = .44$, $d = 0.27$.

We next asked whether infants' familiarization experience affected their later preference for arrays that alternated in numerosity vs. those that were constant in numerosity. We found that of the total amount of time infants looked at the stimuli during the test trials, infants in the Unimodal Familiarization condition spent an average of 47.52% ($SD = 15.09$) looking at the Alternating Number test trials, and 44.15% ($SD = 15.90$) looking at the Constant Number test trials (see Figure 5). In contrast, infants in the Bimodal Familiarization condition spent an average of 41.79% ($SD = 16.37$) looking at the Alternating Number test trials and 45.99% ($SD = 17.02$) looking at the Constant Number test trials. A mixed-design ANOVA with Familiarization Condition (Unimodal or Bimodal Familiarization) as between-subjects factor and Test Trial Type (Alternating Number or Constant Number) as within-subjects factor yielded no significant main effects, but revealed a significant interaction between Familiarization Condition and Test Trial Type, $F(1, 30) = 5.36$, $p < .05$, $\eta^2 = 0.15$. Infants in the Unimodal Familiarization condition looked longer at Alternating Number test trials, whereas infants in the Bimodal Familiarization condition looked longer at Constant Number test trials, even though direct comparisons did not yield statistically significant differences (Unimodal Familiarization: $t(15) = 1.52$, $p = .15$, $d = 0.38$; Bimodal Familiarization: $t(15) = -1.75$, $p = .10$, $d = 0.44$).

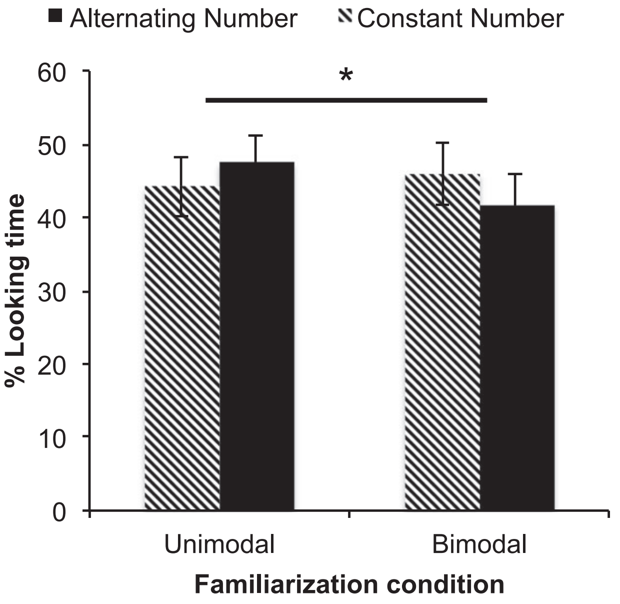

Figure 5 Average percent looking time for Alternating Number and Constant Number test trials in Experiment 2. Error bars reflect standard error of the mean.

Discussion

The results of Experiment 2 show that 6-month-old infants can discriminate frequency distributions from visually presented numerosities, even when the presented numerosities range widely. Infants who were familiarized to a variable frequency distribution of numerosities with a single peak (Unimodal Familiarization condition) looked longer at test sequences of alternating numerosities compared to test sequences containing a constant numerosity. In contrast, infants who were familiarized to a variable frequency distribution of numerosities with two peaks (Bimodal Familiarization condition) looked longer at a constant numerosity compared to alternating numerosities in test.

Note that unlike in Experiment 1, the peak numerosities infants saw in the Bimodal Familiarization condition (12 and 48) did not match the alternating numerosities they saw during test (16 and 36). We purposefully chose the alternating test numerosities such that infants in both familiarization conditions had equal prior exposure to them during familiarization. This choice could have affected infants' looking behavior in the Bimodal Familiarization condition. Previous research has shown that 6-month-old infants can discriminate between ratios that vary twofold (McCrink & Wynn, 2007). Thus, infants in the Bimodal Familiarization condition of the present experiment could have extracted an approximate ratio of 1:4 (12 vs. 48) during familiarization and discriminated it from the ~1:2-ratio (16 vs. 36) in the Alternating Test trials. It seems unlikely that infants in the Bimodal Familiarization condition extracted these ratios, given that they showed a preference for the Constant Test trials over the Alternating Test trials. However, successfully discriminating the ratio of the familiarization peaks (1:4) from the test ratio (1:2) would confirm our prediction that infants can extract distributions from highly variable stimuli.

GENERAL DISCUSSION

In two experiments, we found that 6-month-old infants looked longer at two alternating numerosities after having been familiarized with a frequency distribution of numerosities with a single peak (i.e., a unimodal distribution). In addition, in Experiment 2 we found that infants looked longer at a constant numerosity after having been familiarized with a frequency distribution of numerosities with two peaks (i.e., a bimodal distribution). These findings extend previous work showing that infants can form a representation of a single approximate numerosity from repeated exposures to that numerosity (Brannon et al., 2004; Cordes & Brannon, 2009; Jordan et al., 2008; Libertus et al., 2011; Lipton & Spelke, 2003, 2004; Xu, 2003; Xu & Arriaga, 2007; Xu & Spelke, 2000; Xu et al., 2005; Zosh, Halberda, & Feigenson, 2011). Our results suggest that infants can also form a representation of the frequency distribution of variable numerical input.

Our results show that infants did not merely represent the central tendency of the presented stimuli, but rather that they used the frequencies of the presented numerosities to determine how many distinct distributions were present. In Experiment 1, the central tendencies of the familiarization and test stimuli were numerically indiscriminable from each other (24 in the Unimodal Familiarization condition, 24 in the Constant Number test, 26 in the Bimodal Familiarization condition, 26 in the Alternating Number test). Similarly, in Experiment 2 the central tendencies of the distributions seen during familiarization (26.5 in the Unimodal Familiarization condition and 28.75 in the Bimodal Familiarization condition) were indiscriminable from the central tendencies of the test stimuli (24 in the Constant Number test and 26 in the Alternating Number test). As such, infants' test preferences cannot be explained by simply computing a single running average across the various stimulus presentations.

Our findings also extend previous work showing that infants can extract frequency distributions along a phoneme continuum (Maye et al., 2002) and suggest that infants may be capable of representing frequency distributions across a broad range of stimuli and modalities. One interesting question this raises is how experiencing variability affects thinking, as compared to experiencing little or no variability. A follow-up study to that of Maye and colleagues, using similar stimuli, showed that exposure to a bimodal frequency distribution of speech sounds helped 8-month-old infants to discriminate between difficult sounds, whereas exposure to a unimodal frequency distribution did not aid discrimination (Maye, Weiss, & Aslin, 2008). Paralleling this, a follow-up to the present experiments could examine whether exposure to a bimodal distribution of numerosities enhances infants' numerical discrimination skills, such that, for example, 6-month-old infants could discriminate numerosities that differ by a 2:3 ratio. Previous research has shown that 6-month-old infants typically do not discriminate numerosities differing by a 2:3 ratio (Libertus & Brannon, 2010; Lipton & Spelke, 2003; Xu & Spelke, 2000). However, this discrimination threshold does not seem to be absolute—if numerical information is made redundant across sensory modalities or numerical information is confounded with other perceptual information, infants at this age can indeed discriminate a 2:3 ratio (Baker, Mahamane, & Jordan, 2014; Jordan et al., 2008). Thus, it is possible that familiarization to a bimodal distribution of numerosities could increase the precision in infants' numerical representations, leading to successful discrimination of finer ratios.

Previous research has also shown that infants can discriminate numerosities that are presented in other modalities than vision. For example, 6-month-old infants are able to discriminate between different numbers of tones if they differ by a 1:2 ratio but fail if they differ only by a 2:3 ratio (Lipton & Spelke, 2003). Furthermore, infants are capable of matching a number of tones they hear to the same number of objects in a visual display (Feigenson, 2011; Izard et al., 2009). Thus, future research should explore whether infants' abilities to discriminate single numerosities in the auditory modality also extends to tracking auditory frequency distributions.

Lastly, our results leave open the question which parameters of frequency distributions infants actually represent, and what the limits on their representations might be. For example, our results show that infants represent the general shape of these distributions (unimodal vs. bimodal), but it is unclear whether they retain any information about the variability within the distributions. One way to address this would be to familiarize infants with unimodal and bimodal distributions with a given variability and then test them with unimodal and bimodal distributions with a different variability. Similarly, we always presented infants with symmetrical distributions; it is unclear whether infants also could represent skewed distributions. It also remains open whether infants represent the approximate cardinal values of the number range they experienced. If infants were familiarized to unimodal and bimodal distributions in one number range and then tested with Alternating and Constant Number trials outside of this number range, would infants respond to this change? Finally, there may be limits on the number of distributions infants can concurrently represent. In the present study, we compared distributions with one and two peaks, but infants' real-world experiences may cluster around far more than just two peaks. Could infants represent distributions with three or more peaks? Previous research shows that infants can represent up to three collections, with each collection containing many items, but fail to represent more than three collections at once (Zosh et al., 2011). If infants treat a distribution of approximate numerosities as a single entity, they also might show this signature limit in representing distributions.

In sum, the present study was designed to ask whether infants can represent frequency distributions from variable sequences of numerical information. In Experiment 1, we showed that infants differentiate unimodal distributions containing repeated instances of a single numerosity from bimodal distributions containing alternating presentations of two different numerosities. In Experiment 2, we increased the variability of the numerical input to better reflect dynamic, real-world experience. We found that infants familiarized to a unimodal distribution containing a wide range of numerosities centered around a single mean later preferred looking at two alternating numerosities at test. In contrast, infants familiarized to a bimodal distribution also containing a wide range of numerosities, but with two discriminable peaks, preferred looking at a single numerosity at test. These findings suggest that preverbal infants can represent frequency distributions of variable numerical input and highlight the sophisticated nature of early quantitative competence.

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