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Emergence of the Link Between the Approximate Number System and Symbolic Math Ability

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Experimentally manipulating Approximate Number System (ANS) precision has been found to influence children's subsequent symbolic math performance. Here in three experiments (N = 160; 81 girls; 3–5 year old) we replicated this effect and examined its duration and developmental trajectory. We found that modulation of 5-year-olds' ANS precision continued to affect their symbolic math performance after a 30-min delay. Furthermore, our cross-sectional investigation revealed that children 4.5 years and older experienced a significant transfer effect of ANS manipulation on math performance, whereas younger children showed no such transfer, despite experiencing significant changes in ANS precision. These findings support the existence of a causal link between nonverbal numerical approximation and symbolic math performance that first emerges during the preschool years.

From our earliest days onward, humans think and reason about numerical quantities. Infants—who are years away from acquiring their first number words—represent the approximate numerosity of visual and auditory arrays (Feigenson, Dehaene, & Spelke, 2004). For example, 6-month-old babies habituated to arrays of 16 dots later dishabituate to arrays of eight, with continuous variables like total surface area controlled for (Xu & Spelke, 2000). This sensitivity can be seen even shortly after birth—newborns look longer at arrays of 18 elements when concurrently hearing sequences of 18 sounds, compared to sequences of six, and vice versa (Izard, Sann, Spelke, & Streri, 2009). These intuitive judgments about numerical quantity are supported by an unlearned Approximate Number System (ANS; Feigenson et al., 2004), which has been documented from the start of life across a range of species (Izard et al., 2009; Rugani, Regolin, & Vallortigara, 2011). A key feature of the ANS is that the observers experience less precision in their numerical estimates as the target number grows (e.g., Gallistel & Gelman, 1992). Hence, the ability to discriminate two quantities without counting is ratio-dependent: for example, discriminating 8 dots from 16 is as easy as discriminating 16 dots from 32, and both are easier than discriminating 16 from 24. Observers' accuracy at discriminating approximate numerical quantities gradually increases over early development and then starts to decline in adulthood (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Libertus & Brannon, 2010).

Numerate humans also can determine exactly how many items are in an array, using verbal counting (Feigenson et al., 2004). Unlike ANS representations, the precise integer representations involved in counting allow observers to represent small numerical differences, like that between 50 and 51. Representing integers relies on mastering the logic of linguistic counting—a process that starts early but takes years to complete (Wang & Feigenson, 2019; Wynn, 1990). Thus, while ANS representations are shared among many different species (e.g., Cantlon & Brannon, 2006; Rugani et al., 2011) and do not depend on specific experience (Kanjlia, Feigenson, & Bedny, 2018; Pica, Lemer, Izard, & Dehaene, 2004), integer representations are unique to humans (Matsuzawa, 2009), and require exposure to number words (Pica et al., 2004).

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Despite these phylogenetic and ontogenetic differences, ANS representations and integer representations ultimately are mapped together, as shown by people's ability to use exact integer words to describe the approximate numerosity of an uncounted array (Whalen, Gallistel, & Gelman, 1999). In addition, many studies have observed a relationship between individual differences in people's ANS representations, on the one hand, and their math ability on the other. These studies find that ANS precision correlates with symbolic math performance in children and adults, after controlling for factors such as general intelligence and working memory (Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Schneider et al., 2016 for reviews), even among children at the top levels of math achievement (Wang, Halberda, & Feigenson, 2017). This association does not merely reflect individual differences in visuo spatial processing, as individuals who are congenitally blind exhibit the same degree of ANS-math correlation as sighted individuals (Kanjlia et al., 2018). Finally, the association is predictive: children with more precise ANS representations perform better on future tasks of symbolic math, from a few months to a few years later (Libertus, Feigenson, & Halberda, 2013a, 2013b; Mazzocco, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013; van Marle, Chu, Li, & Geary, 2014). And although some researchers have failed to find evidence of a correlation between ANS precision and math performance (e.g., Price, Palmer, Battista, & Ansari, 2012), meta-analyses suggest that, across the full body of extant studies, there is a reliable relationship between the two ($r = .20$, 95% CI [.14, .26], Chen & Li, 2014; $r = .24$, 95% CI [.20, .28], Schneider et al., 2016).

What is the nature of the relationship between the nonverbal ANS and symbolic math ability? One possibility is that correlations between the two reflect ancillary factors shared between ANS tasks and math tasks, such as executive function abilities (Gilmore et al., 2013). However, studies using stringent controls for inhibitory demands still find a correlation between ANS precision and math, suggesting that they are related above and beyond their shared reliance on executive functions (e.g., Keller & Libertus, 2015; Malone et al., 2019).

Another possibility is that the ANS and the integer representations used in symbolic math are functionally linked. First, it may be that having better symbolic math abilities sharpens the ANS. Studies find that adults with more mathematical education also perform better on nonverbal numerical approximation tasks (Piazza, Pica, Izard, Spelke, & Dehaene, 2013), and longitudinal studies reveal that children's earlier symbolic number understanding significantly predicts their later nonsymbolic numerical performance (Elliott, Feigenson, Halberda, & Libertus, 2019), as well as brain activation during approximate number processing (Suarez-Pellicioni & Booth, 2018). Lastly, children exhibit gains in the precision of their ANS corresponding to the time that they become full-counters, mastering the exact meanings of all of the number words they know (Shusterman, Slusser, Halberda, & Odic, 2016). These findings are consistent with the hypothesis that mathematical experience improves ANS precision, although studies that failed to find effects of symbolic math training on ANS precision suggest that this may happen in a protracted way (Sullivan, Frank, & Barner, 2016). Recent analyses of change in children's and adults' ANS precision suggest that, rather than fine-tuning the underlying numerical representations themselves, age and education might improve ANS performance by increasing observers' ability to focus on the dimension of number rather than on conflicting non-numerical dimensions (Fuhs, McNeil, Kelley, O'Rear, & Vilano, 2016; Piazza, De Feo, Panzeri, & Dehaene, 2018; but see Malone et al., 2019).

Another possibility, not mutually exclusive with the previous, is that having better ANS precision improves math performance. Training studies provide evidence in favor of this account. One approach has been to give adults or children practice mentally adding and subtracting dot arrays. This appears to improve nonsymbolic arithmetic performance in both adults and children, and, critically, to improve their subsequent performance in a symbolic math task relative to participants in a non-numerical control group (Au, Jaeggi, & Buschkuehl, 2018; Park, Bermudez, Roberts, & Brannon, 2016; Park & Brannon, 2013). However, only one of these studies found any improvement in actual ANS precision (i.e., in judging which of two arrays was greater without addition or subtraction, Au et al., 2018), suggesting that the observed math improvements were most likely due to strengthening arithmetic computation abilities, rather than to changes in the ANS itself. Another training approach is to give participants practice deciding which of two arrays is more numerous, without any mental math. One such study found that children who practiced approximate number comparisons later outperformed children who had practiced non-numerical comparisons (involving dimensions like brightness) on a test of symbolic math; again, though, no improvement in ANS precision was observed, leaving it open what mechanism carried this effect (Hyde, Khanum, & Spelke, 2014).

In addition, some research has attempted to experimentally manipulate ANS precision via ANS confidence hysteresis (Odic, Hock, & Halberda, 2014), whereby observers' numerical approximation performance is expected to differ based on the order of trial difficulty. In a series of experiments by Wang, Odic, Halberda, and Feigenson (2016), 5-year-old children completed a series of nonverbal numerical discriminations (rapidly judging whether there were more blue or more yellow dots) in one of two trial orders. Children in the Easy-First condition started with very easy numerical discriminations, in which the numerical ratio between the stimulus arrays was quite large, and gradually progressed to harder ratios. In the Hard-First condition, children started with difficult discriminations and gradually progressed to easier ones. Children in these two conditions saw identical trials; only the order changed. As in Odic et al.'s, original study (2014), children in the Easy-First condition outperformed children in the Hard-First condition, implicating differences in either the precision of children's ANS representations, or in their ability to use the ANS representations to make correct discriminations. Critically, Wang et al. found that children in the Easy-First condition also performed better than children in the Hard-First condition on a symbolic math task immediately afterward. In contrast, there was no performance difference on a subsequent vocabulary task. Additionally, a group of children who completed the ANS task with randomly ordered trials performed at an intermediate level on both the ANS discrimination task and the symbolic math task, suggesting that ANS confidence hysteresis impacts children's performance in both directions (Wang et al., 2016). A nice feature of this study is that all groups of children received equal amounts of experience with numerical tasks —ANS precision was modulated simply through the differences in the ordering of numerical ratios with which children engaged. That this ANS modulation was linked to math performance suggests that there may be a specific link between the ANS and children's symbolic math abilities—though the details of such a mechanism have yet to be determined.

While this body of work provides intriguing evidence regarding the relationship between the ANS and symbolic math, it leaves open several important questions. First, unlike the ANS, which is present from the first days of life (Izard et al., 2009), exact number representations—and the ability to engage in symbolic mathematics—takes years to acquire. When and how do these exact number representations become linked to the developmentally primitive ANS? One possible answer is that the two are linked from the very start of symbolic number use, with the ANS providing a foundation for symbolic mathematics. On this account, experimentally induced modulation of the ANS might affect math performance as soon as children can use number symbols at all. Alternatively, the two systems might become linked in a more protracted way, with ANS modulation only affecting math performance after children have mapped individual number words or symbols to their corresponding ANS representations. This mapping typically occurs between the ages of 4 and 6 years, as children slowly acquire the meanings of number words (Le Corre & Carey, 2007; Libertus, Feigenson, Halberda, & Landau, 2014; Odic, Le Corre, & Halberda, 2015). This account predicts that the transfer effect of ANS manipulation to children's symbolic math performance will not be present at the very start of number symbol use, because number symbols are not yet robustly linked to approximate number representations.

It is also possible, as suggested by Merkley, Matejko, and Ansari (2017), that no mapping at all between the ANS and exact number words is required for ANS experience to affect math performance, because the ANS and math are not directly linked. Their suggestion is that children's emotional stance toward math underlies previously observed effects—that children in the Easy-First condition of experiments by Wang et al. (2016) felt happier or more positive about their number abilities than children in the Hard-First condition, leading them to do better on the math task. This account does not make any explicit predictions about when in development changes in ANS performance might cause such emotional changes, nor when such emotional changes might transfer to math performance. In contrast, the previous account, whereby ANS modulations affect math performance via the mapping between approximate number representations and exact number symbols, does make a clear prediction: no transfer effects should be observed until around 4 years of age, when the mappings become robust.

In addition to the issue of when in development ANS performance becomes linked to math performance, a second open question concerns the duration of ANS manipulation effects. Whereas previous work found that ANS confidence hysteresis was related to performance on a math test taken just minutes later, it remains unknown how long such training effects might last. Here we did not attempt to parametrically vary the time elapsed between ANS manipulation and the test of symbolic math abilities; rather, we took a first step toward this goal by administering the math test 30 min after the ANS manipulation (instead of immediately after, as in Wang et al., 2016). If ANS confidence hysteresis functions similarly to perceptual hysteresis effects, which disappear within seconds (Kleinschmidt, Büchel, Hutton, Friston, & Frackowiak, 2002), we should expect its effects on symbolic math to be similarly short-lived.

In this study, we investigated the developmental trajectory and duration of the causal link between ANS precision and symbolic math performance. We first sought to replicate previous findings of ANS hysteresis and its transfer effects on symbolic math in 5-year-old children. To begin to examine the duration of these effects, we inserted a delay between the manipulation of ANS performance and the test of symbolic math (Experiment 1). Next, in Experiments 2 and 3 we investigated the emergence of the link between the ANS and symbolic math, testing 3- to 5-year-old children to ask whether ANS hysteresis is observed across the entire age range, and whether any observed effects on the ANS transfer to math performance even in the youngest children in our sample.

Participants were recruited by phone or email from the greater Baltimore area, Maryland, USA between January, 2013 and June, 2017. The local population was 65% White and 26% African American. All experiments were conducted in a child development lab on a university campus.

Experiment 1

Method

Participants

Forty children with a mean age of 5 years 3 months participated (SD = 1.9 months, range = 5 years–5 years 7 months; 25 females). Twenty children were randomly assigned to the Easy-First ANS Manipulation condition, and 20 to the Hard-First ANS Manipulation condition. Sample size was based on an a priori power analysis using the *pwr* R package (v1.2-2; Champely et al., 2018). We used the results of Wang et al. (2016) to estimate the likelihood of observing a significant transfer effect of ANS Manipulation on children's symbolic math performance (Cohen's d = 1.12). In this analysis, with an alpha-level of .05 and power = .80, N = 40 was needed to detect a transfer effect of ANS Manipulation condition to Symbolic Math performance.

Parents of all children reported English as the primary language spoken at home. All but one parent in each condition reported having a college degree or higher. Parents identified 32 of the children as White, five as African American, two as Asian, and no racial or ethnic information was provided by the remaining parents. One additional child was excluded for failure to correctly answer more than one of the first three questions (i.e., the easiest three questions) on the Symbolic Math task. Parents provided written informed consent prior to the study, and children received a gift (e.g., t-shirt, book, or toy) to thank them for their participation.

ANS Manipulation Task

Children sat approximately 40 cm from a 13 in. Apple Macbook laptop on which stimuli were presented using a custom Java program. Following the design and procedure of Wang et al. (2016), children saw a sticker of Big Bird on the left side of the screen, and a sticker of Grover on the right. Children were told that they were going to play a computer game in which they would see Big Bird's yellow dots and Grover's blue dots, and should say or point to who had more dots. To reduce spatial conflict, yellow dots always appeared on the left side of the screen and blue dots on the right. The yellow and blue dots appeared simultaneously on each trial and remained visible for 1,200 ms. After children gave a response (either by saying "yellow/Big Bird" or "blue/Grover" aloud, or by pointing), the experimenter immediately pressed the corresponding key on the keyboard ("f" for "yellow," "j" for "blue") to record their responses. Button presses generated trial-by-trial feedback: a prerecorded voice said, "That's right!" following correct responses, and "Oh, that's not right!" following incorrect ones.

The study began with four practice trials. Regardless of ANS manipulation condition, all children saw four randomly selected numerical comparisons of intermediate difficulty (the numerical ratio of these practice trials varied between 1.25 (10 dots vs. 8 dots) and 1.50 (9 dots vs. 6 dots)). The yellow and blue arrays appeared sequentially and each remained visible for 1,200 ms, after which both arrays appeared simultaneously and remained visible for another 1,200 ms. Children then had unlimited time to respond, and received feedback from the computer program. This was done in order to scaffold children toward an understanding of the task. Children averaged 80.63% (SD = 19.19%) correct on these four practice trials, well above chance, $t(39) = 10.09$, $p < .001$, Cohen's $d = 1.60$, suggesting that all children understood the task. Performance on the practice trials did not differ among children in the Easy-First (M = 85%, SD = 17%) and Hard-First (M = 76%, SD = 21%) ANS Manipulation conditions, $t(38) = 1.46$, $p = .152$, Cohen's $d = 0.48$.

After the practice trials children completed 30 test trials, in which the numerical ratio between the dot arrays varied among: 1.11 (i.e., 10 dots vs. 9 dots), 1.14 (8 vs. 7), 1.17 (14 vs. 12), 1.25 (10 vs. 8), 1.50 (9 vs. 6), and 2.00 (10 vs. 5). Each ratio was presented five times with different dot sizes and configurations. The two dot arrays appeared simultaneously and remained visible for 1,200 ms. Children then had unlimited time to respond. The side with the larger number of dots was counterbalanced across trials. To discourage children from relying on cumulative area, on half of the trials the array with the larger number also had more cumulative area (Area Congruent trials), and on the other half the array with the larger number had less cumulative area (Area Incongruent trials).

Children were tested in a pregenerated trial order. Children in the Easy-First condition began with the easiest numerical ratios (i.e., 2.00 and 1.50) and gradually progressed to the hardest ratios (i.e., 1.14 and 1.11). Children in the Hard-First condition completed the same trials in the reverse order—they began with the hardest numerical ratios and ended with the easiest ones. The task took approximately 5 min.

Symbolic Math Transfer Task

A 30-min break followed the ANS Manipulation task. Children returned to the laboratory waiting area, where they were provided with coloring books, puzzles, and toys (none of which involved explicitly numerical content). During the break, neither parents nor the experimenter discussed number or math-related content.

After the break children returned to the testing room and their symbolic math performance was measured using 18 preselected items from Form A of the Test of Early Mathematics Ability, 3rd ed. (TEMA–3, Ginsburg & Baroody, 2003), administered by the same experimenter who had conducted the ANS Manipulation task. The TEMA–3 is a standardized test of informal and formal symbolic math abilities that has been normed for children between 3 and 8 years old. Informal math items test skills that children typically acquire without formal instruction, such as verbal counting and solving spoken math problems using fingers or tokens. In contrast, formal TEMA–3 items test abilities that usually require formal instruction, such as reading and writing Arabic numerals. We selected 18 items that have been previously found to correlate with children's ANS precision (Libertus et al., 2013b), used by Wang et al. (2016). These included items testing informal math abilities (nine items from Numbering, three from Number Comparison, and three from Calculation) and formal math abilities (three items from Numerical Literacy). Numbering assesses basic counting fluency, such as counting backwards. Number Comparison assesses the ability to compare numerals, such as determining whether 7 or 8 is larger. Calculation assesses the ability to solve verbal problems using fingers or tokens, such as how many tokens Joey has if he had one and then got two more. Numeral Literacy assesses fluency in reading and writing numerals. We administered all 18 items, rather than following the standard TEMA procedure of staircasing each child (i.e., continuing until children err on five consecutive items), and administered them to all children in the same order—consistent with Wang et al., 2016. This guaranteed that all the children were tested on the same symbolic math items, such that across conditions, children completed identical math tasks. After each response, children were given neutral positive feedback (e.g., "Ok! Ready for another one?"). Children had unlimited time to answer each question, and were free to skip any question. The task took approximately 15 min. Children's responses were written down by the experimenter and later scored using the TEMA scoring guide (Ginsburg & Baroody, 2003).

Results

Our main question was whether children's overall ANS precision and Symbolic Math performance differed across the ANS Manipulation conditions. On the ANS Manipulation task, children in the Easy-First condition averaged 75% correct, significantly better than children in the Hard-First condition, who averaged 69% correct, $t(38) = 2.13$, $p = .040$, Cohen's $d = 0.67$. This replicated previous findings of ANS confidence hysteresis (Odic et al., 2014: Cohen's $d = 1.36$; Wang et al., 2016: Cohen's $d = 1.18$). Critically, children in the Easy-First ANS Manipulation condition also outperformed children in the Hard-First condition on the Symbolic Math

Transfer task, $t(38) = 2.27$, $p = .029$, Cohen's $d = 0.72$ (Figure 1), suggesting that the group difference in children's ANS performance subsequently transferred to children's symbolic math performance, even after a 30-min delay.

To ask whether the 30-min temporal delay had a significant impact on the transfer effect of ANS Manipulation on children's symbolic math performance, we compared children's Symbolic Math transfer performance in Experiment 1 with the performance of children tested without a delay by Wang et al. (2016). An analysis of variance (ANOVA) with Symbolic Math transfer performance as dependent variable and ANS Manipulation Condition (Easy-First vs. Hard-First) and Delay (30-min Delay vs. No Delay) as independent variables revealed a significant effect of ANS Manipulation Condition, $F(1, 76) = 8.79$, $p = .004$, $\eta_p^2 = .12$, no effect of Delay, and no Delay × Manipulation Condition interaction, Fs < .01, ps > .90. This suggests that the 30-min delay did not significantly impact the robustness of the effect of ANS Manipulation on children's symbolic math performance.

Discussion

Experiment 1 replicated the finding of ANS confidence hysteresis (Odic et al., 2014). We found that 5-year-old children performed significantly better on a numerical approximation task that began with the easiest ratio judgments and gradually progressed to harder ones, compared to the reverse order. Critically, we also replicated and extended the finding that this effect on ANS precision transfers to performance on a symbolic math task

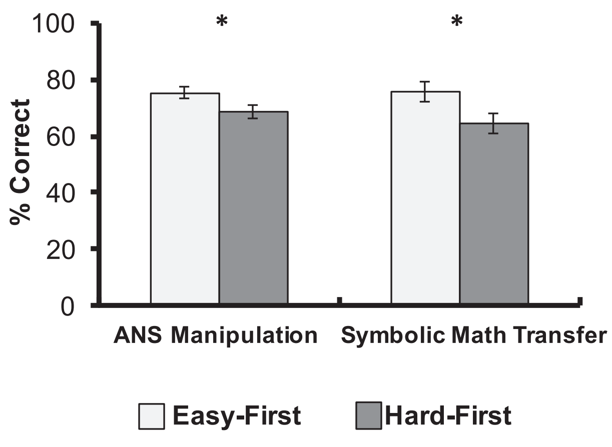

Figure 1. Children's numerical discrimination accuracy and symbolic math performance across Approximate Number System (ANS) Manipulation conditions (Easy-First and Hard-First trial orders) in Experiment 1. Error bars depict ± 1 *SEM*.
*$p < .05$.

requiring exact number representations and number words (Wang et al., 2016). Children who completed the approximate number discrimination trials in the Easy-First order performed better on the math assessment than children in the Hard-First order, even with a 30-min delay between the tasks.

These findings add further support to the idea that there may be a causal link between ANS ability and symbolic math. When in development does this link emerge? Previous research suggests that children do not reliably map ANS representations to exact number symbols (number words) until around 4 or 5 years of age (Le Corre & Carey, 2007; Odic et al., 2015), with the mapping strengthening through the preschool years (Gunderson, Spaepen, & Levine, 2015). It is possible that the ANS does not have a direct impact on children's representation and computation over integers until after this mapping has been formed, in which case we may observe a weaker transfer effect of ANS confidence hysteresis, or no transfer effect at all, in younger children.

In Experiment 2, we examined the consequences of ANS hysteresis across a wider age range than previously tested, in children between 3 and 5 years old. Given recent findings that a scaffolded, Easy-First trial order can enhance the precision of numerical approximation even in preverbal infants (Wang, Libertus, & Feigenson, 2018), we predicted that we would indeed observe ANS hysteresis across our entire age sample. Then we asked whether effects of ANS hysteresis would transfer to children's symbolic math performance. The 5-year-old children in our sample provide an opportunity for a third replication of such a transfer effect (alongside Experiment 1 and Wang et al., 2016). The younger, 3- and 4-year-old children provide a test of whether there are developmental changes in the impact of ANS manipulation on early math performance.

Experiment 2

Method

Participants

A power analysis assuming a small to medium effect size (0.1) suggested that with an alpha-level of .05 and power = 0.80, $N = 80$ was needed to detect an Age × Condition interaction in a regression analysis. Therefore, 80 children with a mean age of 4 years 6 months participated (SD = 6 months; roughly uniformly distributed between 3 years 7 months and 5 years 7 months; 37 females). Parents of all children reported English as the primary language spoken at home. All but three parents in the Easy-First condition and one parent in the Hard-First condition reported having a college degree or higher. Sixty-eight participants self-identified as White, six as African American, two as Asian, and one as belonging to more than one racial or ethnic category; no racial or ethnic information was provided by the remaining parents.

Procedure and Tasks

All aspects of the design and procedure were as in Experiment 1, except that children were tested on the Symbolic Math Transfer task immediately after the ANS Manipulation task (as in Wang et al., 2016). Children were tested with the same numerical ratios in the ANS Manipulation task and the same symbolic math problems as in Experiment 1 (but for age-adjusted tasks, see Experiment 3). To ensure that children across our age range would be equally represented in the Easy-First and Hard-First conditions, we created age-matched pairs. One member of each age-matched pair was assigned to the Easy-First condition and the other to the Hard-First condition. Children in the Easy-First condition averaged 4 years 7 months (SD = 6.5 months) and children in the Hard-First condition averaged 4 years 6 months (SD = 6.5 months), $t(78) = 0.18$, $p = .856$, Cohen's $d = 0.04$. Performance on the practice ANS trials did not differ among children in the Easy-First (M = 80%, SD = 25%) and Hard-First (M = 81%, SD = 25%) ANS Manipulation conditions, $t(78) = 0.22$, $p = .824$ Cohen's $d = 0.05$.

Results

First we asked whether there was evidence of developmental change in ANS precision (Halberda & Feigenson, 2008) or in symbolic math performance (Ginsburg & Baroody, 2003). Collapsing across the two ANS Manipulation conditions, our confirmatory analysis revealed a significant correlation between Age and accuracy on the ANS Manipulation task, $r = .35$, $p = .002$ (Figure 2a), and accuracy on the Symbolic Math task, $r = .68$, $p < .001$ (Figure 2b), confirming developmental improvement. Across the full sample of 3- to 5-year-old children, ANS precision and symbolic math performance were correlated, $r = .40$, $p < .001$ (Figure 2c), as in previous studies (e.g., Libertus et al., 2013a, 2013b), even when controlling for age $r_p = .24$, $p = .033$.

Next we conducted an exploratory analysis to ask whether ANS confidence hysteresis was

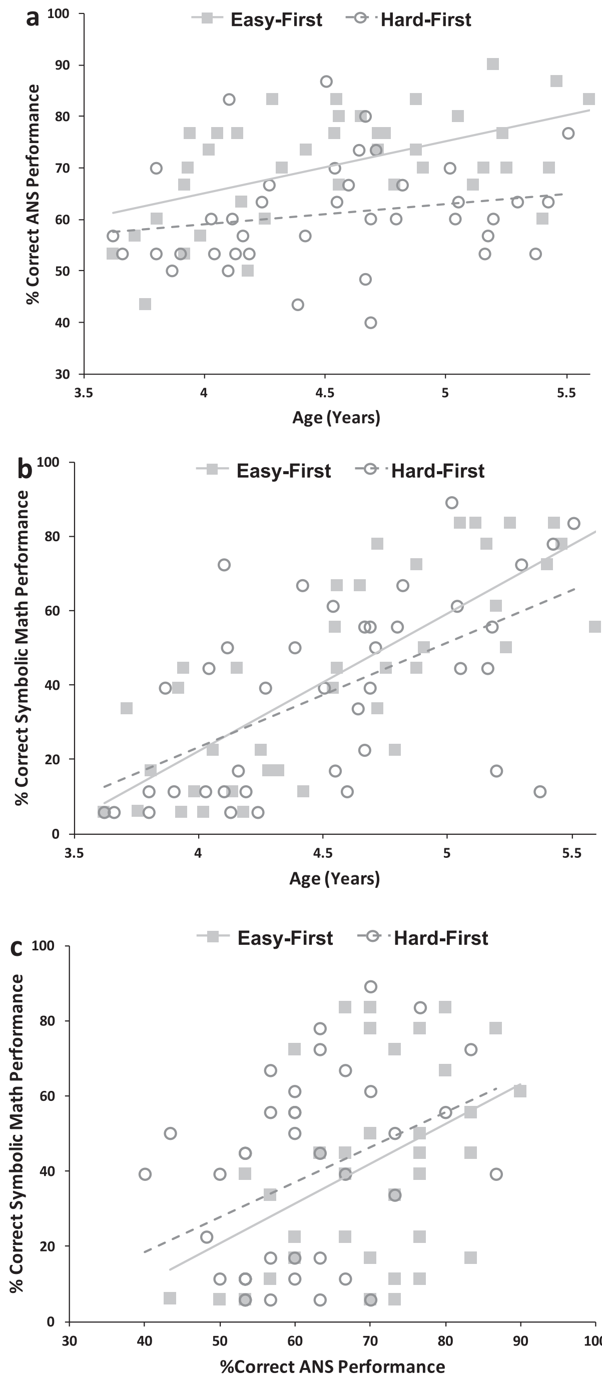

Figure 2. In Experiment 2: (a) children's Approximate Number System (ANS) accuracy as a function of age. (b) Children's symbolic math accuracy as a function of age. (c) Children's symbolic math accuracy as a function of ANS accuracy, split by ANS Manipulation conditions. Fitted lines represent linear model trend lines.

observed across the tested age range, using an analysis of covariance (ANCOVA) with ANS Manipulation Condition as a between-subject factor, Age as covariate, and Accuracy on the ANS Manipulation task as dependent measure. This revealed a significant main effect of ANS Manipulation Condition, $F(1, 76) = 18.31$, $p < .001$, $\eta_p^2 = .19$, with children performing better in the Easy-First than in the Hard-First condition. There was also a significant main effect of Age, $F(1, 76) = 13.24$, $p < .001$, $\eta_p^2 = .15$, with older children outperforming younger children. There was no ANS Manipulation Condition × Age interaction, $F(1, 76) = 2.27$, $p = .136$, $\eta_p^2 = .03$. These results suggest that ANS confidence hysteresis was observed across the sample of 3- to 5-year-old children, with no significant change in its strength across these ages.

Our key question was whether there were any age-related changes in the extent to which changes in children's ANS precision transferred to their symbolic math performance. An ANCOVA with ANS Manipulation Condition as between-subject factor, Age as covariate, and Accuracy on the Symbolic Math Transfer task as dependent measure revealed no significant effect of ANS Manipulation Condition, $F(1, 76) = 1.09$, $p = .300$, $\eta_p^2 = .01$, a significant main effect of Age, $F(1, 76) = 67.21$, $p < .001$, $\eta_p^2 = .47$, and no ANS Manipulation Condition × Age interaction, $F(1, 76) = 1.21$, $p = .275$, $\eta_p^2 = .02$. An inspection of the data suggests that the large effect of Age and the null effect of ANS Manipulation Condition may have been due to younger children's poor performance on the Symbolic Math Transfer task—children younger than 4.5 years old were correct on < 30% of the Symbolic Math questions. This was true regardless of whether children had just completed the ANS task in the Easy-First or Hard-First order. If the Symbolic Math task was too hard for the younger children in our sample, we would be unlikely to find age differences on the transfer effect of the ANS Manipulation task—even if such an effect were actually present. Indeed, since our observed effect size (0.02) was smaller than the assumed effect size of 0.1 from the power analysis, the current experiment was likely underpowered to detect a significant Age × Condition interaction.

A cursory look highlights the difficulty that the younger children in our sample had completing the symbolic math task, regardless of ANS Manipulation condition. To ask whether children's performance on the ANS Manipulation and Symbolic Math Transfer tasks differed on a group level across ages, we performed a median-split by age: children in the younger age group ranged between 3 years 7 months and 4 years 6 months (M_{age} = 4 years 1 month, SD = 3 months), and children in the older age group ranged between 4 years 7 months and 5 years 7 months (M_{age} = 5 years, SD = 3.7 months). This exploratory analysis revealed that in the older group, children in the Easy-First versus Hard-First ANS conditions performed differently in both ANS accuracy, $t(38) = 4.50$, $p < .001$, Cohen's $d = 1.42$, and Symbolic Math performance, $t(38) = 2.14$, $p = .038$, Cohen's $d = 0.68$, replicating the results of Experiment 1 and Wang et al. (2016). On the ANS Manipulation task, these older children in the Easy-First condition performed 75% correct on average (SD = 8%), and children in the Hard-First condition performed 63% correct (SD = 10%). On the Symbolic Math Transfer task, older children in the Easy-First condition on average performed 63% correct (SD = 18%), and children in the Hard-First condition performed 48% (SD = 24%; Figure 3). This is consistent with our predictions, and consistent with the results of Experiment 1.

In the younger group, children in the Easy-First ANS Manipulation condition performed 66% correct on average (SD = 12%), and children in the Hard-First condition performed 60% correct (SD = 11%), $t(38) = 1.82$, $p = .076$, Cohen's $d = 0.57$. As hypothesized, there was no significant difference in these younger children's Symbolic Math performance: children in the Easy-First condition performed 23% correct (SD = 16%), and children in the Hard-First condition performed 28% correct (SD = 23%), $t(38) = -0.88$, $p = .385$, Cohen's $d = 0.28$ (Figure 3). Combined with the significant effect of Condition among the older group of children in our sample, this is consistent with the age effect that was anticipated. However, as mentioned earlier, the lack of significant effect of Condition on younger children's Symbolic Math performance is hard to interpret. If younger children experienced a weaker effect of ANS confidence hysteresis than older children, they may also have experienced a weaker math transfer effect (although this study lacks sufficient power to detect a significant Age × Condition interaction). Alternatively, or in addition, it could be that task difficulty (which was originally chosen for the 5-year-olds in Experiment 1) was too great for the younger children in Experiment 2, making it hard to observe significant effects. We address this directly in Experiment 3.

Finally, we explored the effects of non-numerical stimulus features on children's performance. On half of the trials the numerically larger array had more cumulative area, and half the time it had less. This gave us an opportunity to test one possible mechanism of ANS performance modulation: change in the ability to focus on the dimension of

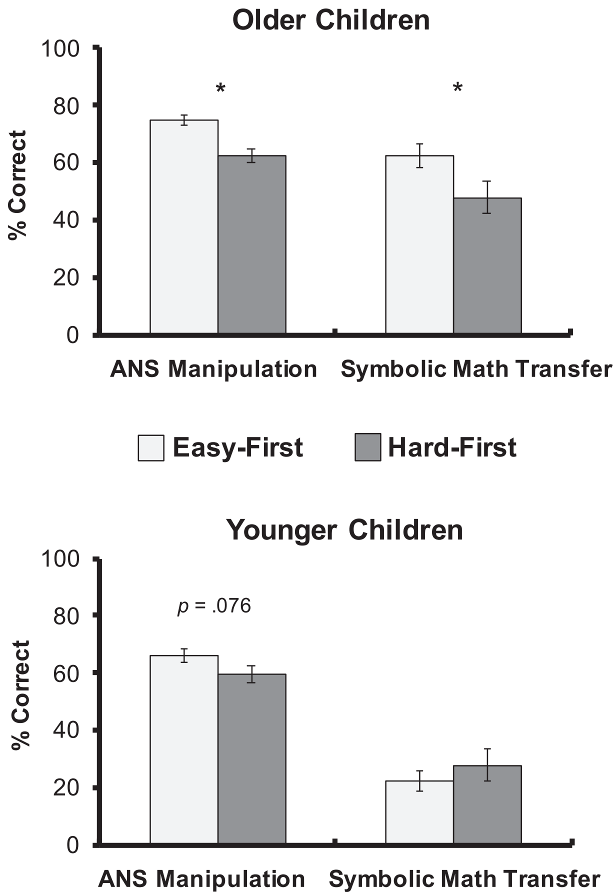

Figure 3. Approximate Number System (ANS) manipulation and symbolic math accuracy in Experiment 2 as a function of ANS Manipulation condition for children older than 4.5 years and children younger than 4.5 years. Error bars indicate ± 1 *SEM*.
*$p < .05$.

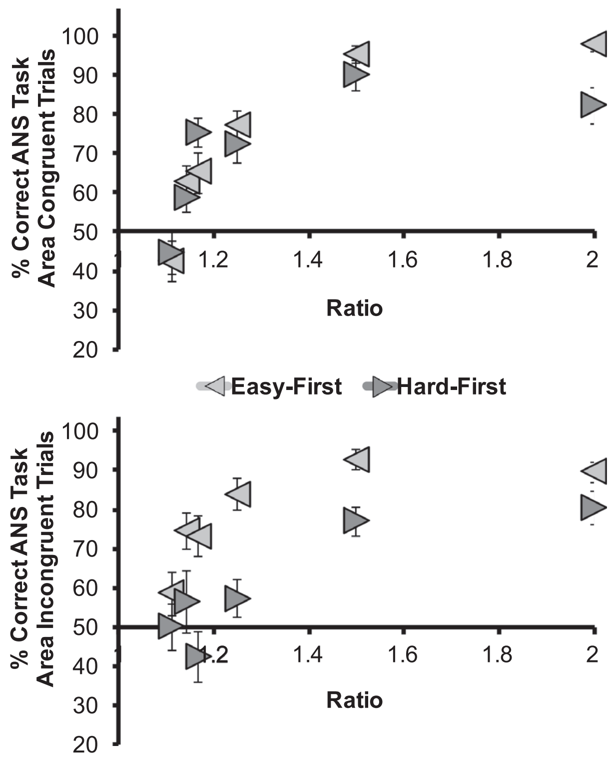

Figure 4. Approximate Number System (ANS) manipulation accuracy in a combined sample of children from Experiment 1 and the children older than 4.5 years from Experiment 2, separated by ANS Manipulation condition (Easy-First vs. Hard-First) and Area condition (Area Congruent vs. Incongruent).

numerosity and ignore continuous variables (Fuhs et al., 2016; Piazza et al., 2018). We therefore combined children's performance in Experiment 1 with performance from the children older than 4.5 years in Experiment 2 (yielding a total sample of $N = 80$). We then ran a 6 (numerical ratio) × 2 (area: congruent vs. incongruent) × 2 (ANS Manipulation condition: Easy-First vs. Hard-First) repeated measures ANOVA with accuracy as the dependent measure. This revealed a significant main effect of Ratio, $F(5{,}390) = 43.88$, $p < .001$, $\eta_p^2 = .56$, with higher accuracy on easier ratios, consistent with the psychophysical signature of the ANS. There was a significant main effect of ANS Manipulation Condition, $F(1,\ 78) = 21.36$, $p < .001$, $\eta_p^2 = .27$, and no interaction between ANS Manipulation Condition and Ratio, $F(5{,}390) = 0.76$, $p = .579$, $\eta_p^2 = .01$. In addition, there was no main effect of Area, $F(1,\ 78) = 1.09$, $p = .300$, $\eta_p^2 = .01$, suggesting that children performed similarly on trials where number and area were congruent and trials where number and area were incongruent. However, there was a significant interaction between Area and ANS Manipulation Condition, $F(1,\ 78) = 15.75$, $p < .001$, $\eta_p^2 = .20$. The difference in children's performance between the Easy-First and Hard-First ANS manipulation conditions was more pronounced on Area Incongruent trials than Area Congruent trials. There was also a significant Ratio × Area Control × ANS Manipulation Condition interaction, $F(5,\ 390) = 3.39$, $p = .005$, $\eta_p^2 = .04$. On Area Congruent trials, the effect of ANS Manipulation Condition was the strongest for the easiest Ratios, whereas on Area Incongruent trials the effect of Condition was stronger for the harder Ratios (Figure 4). These results suggest that hysteresis may have helped children in the Easy-First condition to better focus on the numerical dimension of the stimuli and filter out the conflicting representation of cumulative area as the task progressed, whereas children in the Hard-First condition lacked this benefit.

Discussion

In Experiment 2 we found that ordered scaffolding of numerical approximation trials affected ANS performance in children ranging from 3 to 5 years old. Combined with findings of similar ANS confidence hysteresis in 6-month-old infants (Wang et al., 2018), these results suggest that ANS representations can undergo temporary modulation even in children with limited self-awareness of their own numerical abilities, and with no formal mathematical education. However, Experiment 2 also revealed that whereas changes in ANS precision were linked to superior symbolic math performance in older children (i.e., above 4.5 years), they seemed to have no such transfer effect in younger children. One interpretation of this developmental difference is that the 3- and 4-year-olds in Experiment 2 had not yet formed a robust mapping between ANS representations and exact number symbols, or were in the early stages of the mapping process, and that having a robust mapping is necessary for ANS precision to impact symbolic math performance. However, an alternative explanation is that the symbolic math task was just too challenging for younger children; their performance was too close to floor to allow any potential transfer effects of ANS hysteresis to emerge. As noted earlier, all children in Experiment 2, regardless of age, were tested on the same ANS ratios and symbolic math problems, which were originally designed for children who were approximately 5 years old (Experiment 1; Odic et al., 2014; Wang et al., 2016). Therefore, it is possible that younger children would exhibit a more robust ANS confidence hysteresis effect and a significant transfer effect to symbolic math performance if their overall performance on both tasks were roughly equivalent to that of the older children.

To test this possibility, in Experiment 3 we tested younger children in an easier version of the ANS Manipulation task and the Symbolic Math Transfer task. Items within each task were chosen to roughly equate the predicted performance of 3.5- to 4.5-year-old children to that of the 4.5- to 5.5-year-old children in Experiment 2.

Experiment 3

Method

Participants

Forty children with a mean age of 4 years participated (*SD* = 3 months; range = 3 years 7 months–4 years 6 months; 19 females). Parents of all children reported English as the primary language spoken at home (100%). All but one parent reported having a college degree or higher. Thirty-two participants were identified by their parent as White, four as African American, one as Asian; no racial or ethnic information was provided by the remaining parents.

Procedure and Tasks

All aspects of the experimental design and procedure were as in Experiment 2, except that the ratios in the ANS Manipulation task and the items in the Symbolic Math Transfer task were adjusted for younger children. In the ANS Manipulation task, the hardest numerical ratio (1.11) was replaced by a much easier ratio of 3.00, in order to yield overall accuracy level similar to that of the older group of children in Experiment 2 (see Odic, Libertus, Feigenson, & Halberda, 2013 for justification of ratio choices). Hence, the numerical ratio between the dot arrays varied among: 1.14 (8 vs. 7), 1.17 (14 vs. 12), 1.25 (10 vs. 8), 1.50 (9 vs. 6), 2.00 (10 vs. 5), and 3.00 (15 vs. 5). Performance on the practice ANS trials did not differ among children in the Easy-First (M = 63%, SD = 21%) and Hard-First (M = 70%, SD = 20%) ANS Manipulation conditions, $t(38) = 1.19$, $p = .242$ Cohen's $d = 0.38$.

In the Symbolic Math Transfer task, 12 questions within the expected ability range of typically developing 4-year-old children were selected from the same Informal and Numeral Literacy categories of the TEMA–3. Eight were in the Numbering category, two in the Number Comparison category, one in the Calculation category, and one in the Numeral Literacy category.

Results and Discussion

Confirmatory analyses revealed that, as shown in Figure 5, children in the Easy-First ANS Manipulation condition performed significantly better (71%, SD = 13%) than children in the Hard-First condition (62%, SD = 9%), $t(38) = 2.27$, $p = .029$, Cohen's $d = 0.72$, replicating the ANS confidence hysteresis effect previously discovered. However, there remained no significant difference in children's performance on the Symbolic Math Transfer task among children in the Easy-First ANS Manipulation condition (53%, SD = 18%) and the Hard-First condition (49%, SD = 19%), $t(38) = 0.56$, $p = .581$, Cohen's $d = 0.18$, despite the fact that children's overall symbolic math performance was comparable

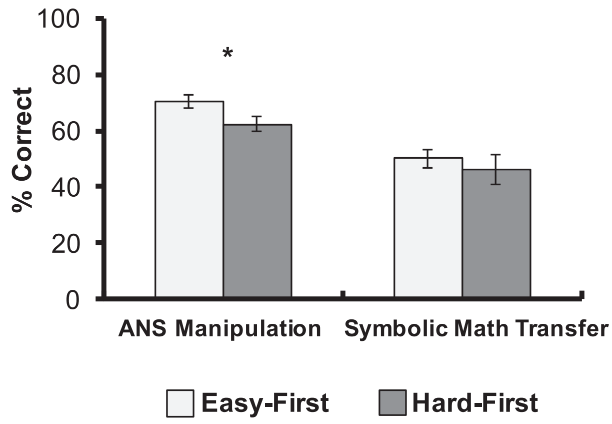

Figure 5. Approximate Number System (ANS) manipulation and symbolic math accuracy in Experiment 3 as a function of ANS Manipulation condition. Error bars indicate ± 1 *SEM*.
$^*p < .05$.

to that of the older children in Experiment 2, $t(78) = 0.97$, $p = .338$, Cohen's $d = 0.25$.

To further ask whether ANS confidence hysteresis exerted a significant symbolic math transfer effect in these younger children, we compared the results from children in Experiment 3 with those of the younger than 4.5-year-old children in Experiment 2. First we compared their performance in the ANS Manipulation task. A 2 (ANS Manipulation Condition) × 2 (Experiment) ANOVA with ANS accuracy as the dependent measure revealed a significant main effect of ANS Manipulation Condition, $F(1, 76) = 8.37$, $p = .005$, $\eta_p^2 = .10$, no main effect of Experiment, $F(1, 76) = 1.91$, $p = .171$, $\eta_p^2 = .02$, and no Condition × Experiment interaction, $F(1, 76) = 0.11$, $p = .743$, $\eta_p^2 = .001$. Hence 3- and 4-year-olds in Experiments 2 and 3 experienced a similar ANS confidence hysteresis effect, even though children in Experiment 2 experienced more difficult numerical discriminations and more difficult symbolic math problems. We next compared their symbolic math performance. A 2 (ANS Manipulation condition) × 2 (experiment) ANOVA with Symbolic Math accuracy as the dependent measure revealed no main effect of ANS Manipulation Condition, $F(1, 76) = 0.06$, $p = .800$, $\eta_p^2 = .001$, a significant effect of Experiment, $F(1, 76) = 34.54$, $p < .001$, $\eta_p^2 = .31$, and critically, no ANS Manipulation Condition × Experiment interaction, $F(1, 76) = 1.04$, $p = .311$, $\eta_p^2 = .014$. This suggests that although children, not surprisingly, performed better with the easier symbolic math problems used in Experiment 3, there was no transfer effect to symbolic math in either experiment. Hence even when overall performance between older and younger children was more closely aligned, improvements in ANS performance changed math performance in 5-year-olds, but not in younger children.

General Discussion

Research of the past decade has suggested a surprising link between the evolutionarily endowed ability to represent approximate numerosities, on the one hand, and symbolic mathematics abilities on the other (Feigenson et al., 2013). Recent efforts to examine the nature of this link have begun to make progress, in part by showing that experimentally induced changes to children's ANS performance can cause changes in math performance (e.g., Hyde et al., 2014; Park & Brannon, 2013; Park et al., 2016). However, it remains unclear how and when this link emerges developmentally, as well as the duration over which modulations of the ANS can affect math performance. In this study, we aimed to replicate the effect of one particular type of ANS modulation—ANS confidence hysteresis—while also exploring possible developmental changes in the effect, as well as its temporal robustness.

In Experiment 1, we found that 5-year-old children performed better on an approximate numerical discrimination task when the trials started with easy discriminations and gradually progressed to harder ones (Easy-First), compared to the reverse order (Hard-First). This suggests that children's approximate number performance can be affected by immediate experience, rather than operating with fixed precision (see also Jordan, Suanda, & Brannon, 2008; Odic et al., 2014; Wang et al., 2016, 2018). In addition, we found that this modulation affected 5-year-olds' symbolic math: children who had performed the Easy-First approximate numerical discriminations also performed significantly better on a portion of the TEMA–3 math test, even after a 30-min delay. These findings replicate and extend previous work (Wang et al., 2016).

In Experiments 2 and 3, we used a cross-sectional approach to examine the possibility of developmental change in this effect. We observed robust ANS confidence hysteresis in 3- to 4-year-old children, replicating the effect previously seen in 4- to 5-year-olds (Experiment 1; Odic et al., 2014; Wang et al., 2016). Our analyses of the impact of the congruity of number and area on children's performance are consistent with the suggestion that confidence hysteresis might impact children's ANS precision in part by leading them to filter out competing information from non-numerical dimensions (Piazza et al., 2018). Together, the current results and previous findings suggest a reframing of our understanding of ANS precision and its development: in addition to a stable "trait" that is fixed at a given age for a given observer, ANS precision can be thought of as a dynamic "state" that responds to the environment (see Jordan et al., 2008). It may also be that these dynamic changes occur at the level of decision making using ANS representations (e.g., when gathering evidence to make a decision about numerosity), rather than reflecting a direct sharpening of the underlying ANS representations themselves (Piazza et al., 2018). Much remains to be discovered concerning this state-like change of ANS precision.

In Experiments 1 and 2 we also replicated the finding that experimentally manipulating ANS performance affects symbolic math performance in children older than 4.5 years. However, despite exhibiting significant modulations in their ANS precision, children younger than 4.5 years did not show differences in their symbolic math performance following the ANS manipulation (Experiments 2 and 3), even when the tasks were made easier to roughly match difficulty across ages. The lack of a transfer effect in younger children suggests that the link between the ANS and symbolic number representations undergoes important developmental change between the ages of 3 and 5 years. The null effect of ANS manipulation on younger children's symbolic math performance can potentially be thought of as a control for domain-general accounts of the observed effects in older children. If domain-general factors such as self-confidence, which has been shown to be malleable in children as young as 3 years of age (Dowsett & Livesey, 2000; Ekeland, Heian, & Hagen, 2005), were responsible for the effect of ANS manipulation on symbolic math performance (as suggested by Merkley et al., 2017), we might have expected children to respond similarly across our entire age range. That they did not, coupled with the previous finding that ANS manipulation had no transfer effect on 5-year-old children's vocabulary performance (Wang et al., 2016), suggests that the effect of ANS manipulation on math performance may be specifically numerical in nature.

The nature of the developmental change we observed merits further investigation. One possibility raised earlier is that children need to build a robust mapping between ANS representations and integer representations in order for changes in ANS performance to affect symbolic math. If children lack a robust mapping until after 4 years old (Le Corre & Carey, 2007; Odic et al., 2015), then changes to the ANS have no means of affecting math performance in these younger children (Wong, Ho, & Tang, 2016). Once children have a mapping in place, the ANS could plausibly be used to generate ball-park estimates for correct answers to symbolic problems (Gilmore, McCarthy, & Spelke, 2007), in which case modulations of ANS precision could influence children's accuracy or efficiency at solving symbolic problems (by this or some other mechanism). Future research could test this possibility by investigating the influence of ANS confidence hysteresis on children's error patterns in symbolic math problems.

It is also possible that developmental change in metacognition contributed to the age differences we observed. In previous work we showed that the effect of ANS manipulation on symbolic math performance was unlikely to have been caused by changes in domain-general factors such as overall motivation or mood, because changes to the ANS did not change children's performance on a verbal task (Wang et al., 2016). But the transfer effect could instead involve domain-specific meta-cognitive factors, such as children's confidence in their numerical skills (Baer, Gill, & Odic, 2018; Vo, Li, Kornell, Pouget, & Cantlon, 2014). That is, ANS confidence hysteresis might affect children's internal sense of their ability to reason about quantities. For example, children who experienced the Easy-First ANS Manipulation might have been more likely to think of themselves as mathematically competent, thereby leading to more perseverance in subsequent symbolic math problems. This could also potentially explain the more lasting effect of ANS confidence hysteresis on symbolic math performance, compared to the transient effects of perceptual hysteresis. If children's domain-specific sense of confidence changes with age, this could contribute to the developmental change observed in this study.

It is important to note that the cross-sectional approach we used here to examine developmental change in ANS confidence hysteresis and its effects on math performance is only one of many possible approaches, and future work using other study designs will be important to strengthen and extend our findings. For example, our study lacked a baseline measure that could have ensured that groups did not differ in ANS precision prior to our manipulation. Given that the phenomenon of ANS confidence hysteresis has now been observed in at least seven experiments (two by Odic et al., 2012, one by

Wang et al., 2016, one by Wang et al., 2018, and three in the present series), it seems unlikely that flukes in group assignment caused the systematically better performance by participants who completed numerical discriminations in an Easy-First trial order. Still, testing participants both before and after the experimental training experience could further bolster our conclusions. In addition, future work may wish to more closely examine the scope of ANS hysteresis effects in transfer tasks. In previous research, ANS hysteresis in 5-year-olds was linked to better performance in a subsequent symbolic math task, but not a subsequent verbal task (Wang et al., 2016). Whether ANS hysteresis transfers equally to multiple kinds of mathematical processing (e.g., geometrical reasoning) will be an interesting question for future work, which could also test more types of non-numerical reasoning (besides verbal ability) to more fully map out cases in which ANS improvements affect later processing versus cases in which they do not.

The present finding that modulation of the ANS changes mathematical performance in older children but not younger ones suggest that the link between numerical approximation and symbolic math undergoes development in early childhood. But more broadly, the wealth of recent research investigating the relationship between these two aspects of numerical thought suggests that their link may not be simple, and is unlikely to be described along a single dimension. Instead, the ANS and symbolic mathematics may be linked in multiple ways—from direct links between ANS representations and symbolic mathematical thought (Park & Brannon, 2013; Wang et al., 2016) to changes in emotions or attitudes toward math that result from ANS differences (Braham & Libertus, 2018; Merkley et al., 2017). These links may also be seen at multiple time-scales—from immediate effects like those observed here, to longer term effects whereby individual differences in the ANS affect individuals' interest in and choice to engage with math (Libertus, 2019). Characterizing this space of influence may yield important insights for mathematical instruction, but more immediately will help shed light on the relationship between an evolutionarily ancient system for representing quantities and the formal mathematics that only humans ever acquire.

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Appendix

Table A1

Average Percent Correct (SE) of Children's Symbolic Math Performance in Experiments 2 and 3

TEMA category	Numbering	Number comparison	Numeral literacy	Calculation
Experiment 2 (older children)				
Easy-First	72.78 (3.74)	71.67 (8.12)	48.33 (7.83)	36.67 (7.98)
Hard-First	56.67 (5.52)	50 (8.55)	36.67 (7.98)	31.67 (6.15)
Experiment 3 (younger children with easier problems)				
Easy-First	63.89 (4.41)	20 (7.61)	25 (9.93)	10 (6.88)
Hard-First	59.44 (4.29)	20 (7.61)	20 (9.18)	5 (5)